Non-decreasing Sequences

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# Non-Decreasing Sequences

**Alexander Bruno and Amy Klass**

**Advisor:** Dr. Jon Beagley

## Abstract

Non-decreasing sequences are a generalization of binary covering arrays, which has made research on non-decreasing sequences important in both math and computer science. The goal of this research is to find properties of these non-decreasing sequences as the variables $d$, $s$, and $t$ change. The goal is also to explore methods for creating a maximum length non-decreasing sequence for a given strength and size set. Through our research, we discovered and proved basic properties of these non-decreasing sequences. In addition to this, we can describe a method we used while trying to find the maximum length of a sequence.

## Definitions and Notation

- **Let $S$** be a set of $s$ elements
- The **strength** of non-decreasing sequence is the amount of subsets whose union we consider, and is represented using $d$
- A **non-decreasing sequence** of strength $d$ is a sequence of non-empty subsets, $\{S_1, S_2, \ldots, S_t\}$, where the union of any $d$ previous subsets does not contain any subsequent subset
- The number of subsets in a non-decreasing sequence is called the **length**, $t$
- $NDS(d, s, t)$ is the set of non-decreasing sequences with strength $d$, $s$ elements and length $t$
- **NDST** ($d, s$) is the maximum $t$ such that $NDS(d, s, t)$ is non-empty
- Let $r_j$ be the number of elements in the subset $S_j$

## Binary Arrays

- Represent a non-decreasing sequence using an $s \times t$ binary array
- Rows represent elements of $S$
- Columns represent subsets of non-decreasing sequence

## Basic Results

**Theorem 1** - Permuting rows in a binary array gives another $NDS(d, s, t)$.

**Theorem 2** - If the union of any $d$ subsets contain all elements in $S$, no subsets can be added to the sequence.

**Theorem 3** - Every subset in $NDS(d, s, t)$ must be distinct for $d \geq 1$.

**Theorem 4** - $NDS(d, s, t) \subseteq NDS(d, s + 1, t)$

**Corollary 5** - $NDST(d, s) \geq k \cdot NDST(d, s)$, where $k \in \mathbb{Z}$.

## Standard Sequence

**Theorem 6** - There exists an $NDS(d, s, t)$ where the first $s$ subsets are of size 1. We call this a **standard non-decreasing sequence**.

## Bounds

- **Gives range for** $NDST(d, s)$
- **Lower bound** is the length of sequence constructed for a given $d, s$
- **Upper bound** initially $2^s - 1$, number of nonempty subsets possible for any set $S$ with $s$ elements
- **Upper bound decreased using Theorems 7, 8, and 9**

<table>
<thead>
<tr>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>$2^s$ - 1</th>
</tr>
</thead>
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<td>1</td>
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<tr>
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<td>127</td>
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</tbody>
</table>

Table 1. Bounds for $d = 2$

## Future Work

- Find exact formula for $NDST(d, s)$
- Find different computational methods
- Find relation to binary covering arrays
- Effect of permuting columns
- Find bounds for larger $d$ and $s$ values

## References


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