Abstract

Our goal for this project was to expand and improve upon the findings of George I. Bell and Nicholas Fonseca, who have both written papers on optimization in Chinese Checkers. While their work focuses mainly on cooperative games between two players, and three players, we have considered games for six players. While doing this, we have redefined the playing board in a more intuitive manner, while developing and proving its associated distance formula. As well, we have conjectured the shortest game for six players, and are working to generalize a formula for the number of moves required to finish a six player game as fast as possible. This could further incite research to generalize a lower bound for any number of players.

Definitions

- **Cell** - Position in which a marble may be placed.
- **Move** - Changing the marble’s position from one cell to another. This may be done in three ways:
  - **Step** - Moving a marble to an adjacent cell. An adjacent cell may be horizontal or diagonal, but not vertical.
  - **Jump** - Moving a marble by hopping over another marble of any army into an empty cell. This does not capture the other marble, and jumps are not compulsory.
  - **Chain** - More than one jump in a single turn by the same marble.
- **Goal** - Opposing end, ending position for an army of marbles.
- **Man** - Single marble owned by a player. This is part of their army.
- **Rung** - Single move by a player in a round of turns.
- **Round** - All of the players take one turn.
- **Movement Structure** - A movement structure allows a player to move along a chain. There is one defined movement structure.
- **Ladder** - A ladder of marbles that allow a player to move as a chain.
- **Jump** - A single marble that is part of a ladder.

Graphical Representation

Previous Board Design

- **Cartesian Board**
- **Movement constricted**
- **No movement along y=x**
- **Chess notation**
- **More difficult for more players**
- **Semi-intuitive movement**

Our Board Design

- **Cartesian Board overlaid playing board**
- **Holes in the board (e.g. b12 does not exist as a cell)**
- **Modified chess notation**
- **Any number of players**
- **Intuitive movement**
- **Identified player bases A-F**

Distance Formula

\[
\text{Distance} = \begin{cases} 
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} & \text{if } |x_1| > |y_1| \\
\frac{1}{2} |(x_2-x_1)| & \text{if } |x_1| \leq |y_1|
\end{cases}
\]

As previous authors have provided a distance formula for their coordinate system, we had to develop our own. Proof.

We know that the x-coordinates are spaced by 2 while the y-coordinates are spaced by 1. We define the x-distance to be |x| and the y-distance to be |y|. Because the rules of Chinese Checkers allow a player to move directly horizontal and/or diagonal, we have four cases to prove:

- **Case 1**) Strictly horizontal movements: No vertical moves mean that |y| = 0 and |x| > |y|. Because the x-coordinates are spaced by 2, the number of horizontal steps, n, must be half of the x-coordinates, say n = |x|/2. We may then manipulate n = |x|/2 + ½|x| = |y| + ½|x| - 1 |y|.

- **Case 2**) Diagonal movements along a single diagonal: Because the piece would be moving diagonally, the x-coordinate movement is spaced by 1 since it falls between a strict horizontal movement (of 2 coordinates). Hence, |x| = |y|. Thus, the number of steps would be n = |x|.

- **Case 3**) Diagonal movements one a single diagonal with horizontal movements: This case can be broken up into two cases previously proven. Because the diagonal movement is on the same diagonal, we apply Case 2, allowing this movement to be described as |y|. As with Case 3, we can include horizontal movement as ½|x| + |x| + ½|x| = |x| + ½|x| = |x| again.

- **Case 4**) Diagonal movements of different diagonals: Diagonal movement of different diagonals may be expressed as two distinct movements. Note that when considering a marble at (0,0), the first move is not necessary. First, the marble moves along a diagonal from (0,0), or on our graph, 112. This movement will place the marble along a vertical line to (x,y). This movement is shown to be |x| by Case 2. Then, the marble may move in a zigzag pattern of one step diagonal movements. When the marble zags, it will return to its previous position. Note that these zigzags do not need to be strictly consecutive, but only that each zigzag needs an associated z Sag. This second motion may be defined as |y| - |x|, as we have already moved x a distance. Thus, the full movement is |x| + |y| - |x|.

Search for the Lower Bound

Absolute Lower Bound: \(9p + 1\), where \(p\) is the number of players

- Assume winning player moves first
- All marbles move 9 times
- This requires 9 rounds and 5 additional turn, thus 9 moves for each player, and an additional turn for the winning player

Impossible to put piece in goal on first move, so more moves necessary

Realistic Lower Bound: \(9p + c - 1\), where \(c\) is the number of moves it takes to build a successful chain

Successful chain - a chain that allows the winning player to move a marble from a cell in its starting base to a cell in the goal

Ladder options:
- 5 rung option (10 steps or 5 jumps)
  - Moves piece from row to row now
  - Requires building 5 rungs
- 6 rung option (12 steps or 6 jumps)
  - Moves piece from row to row now
  - 2 preexisting rungs from base and goal
  - Requires building 4 rungs

6-Player Game

Our formula for the 6 player game provides a realistic lower bound of 63, as \(c = 8\). We have found an example of a 66 move solution, and we conjecture the existence of a 64 move solution. Our current solution follows the current structure:

- Assume Player A is winning player and Player B moves first, moving clockwise
- Players B and C build ladders for blue and red locations
- Built using green and yellow locations as rungs
- Players D and E build ladders for green and yellow locations
- Built using blue and red locations as rungs
- Player D emplaces the order in 4, 1, 6, 5, 2, 7, 5, 10, 8
- This order follows the base groupings mentioned in Location Types
- Player A fills its goal in the order 1, 6, 4, 2, 7, 9, 5, 8, 10
- The grouping order is the same as for Player D
- Player A emplaces their base in the order 5, 6, 1, 4, 7, 2, 8, 3, 10
- This follows roughly the same grouping order as its goal
- Location: 5 is emplaced first in order to fulfill a mandatory move
- Least disruptive to jump a group of one piece

Location Types

- **Four distinct locations (as shown by color)**
- **Cannot jump between locations**
- **Bell calls these locations "man types" 0-3**
  - 0-Blue, 1-Green, 2-Yellow, 3-Red
  - **Bases and their respective goals have equivalent location types**
  - Within base (see image): 1A, and 6 are the same type, 7 and 9 and are the same type, 3, 8, and 10 are the same type
  - **Ladder Building Rules**
  - Blue and Red may jump over Green and Yellow
  - Green and Yellow may jump over Blue and Red

References