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The Negotiator's Role in a Buyer-Seller Game

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Abstract

In game theory, buyer-seller games rarely utilize a negotiating third party. Any negotiations are typically conducted by the buyer and seller. This study, motivated by the real estate market, uses sequentially and simultaneously played game models to explore the influence a self-interested, negotiating, third party has on player payoffs. For the sequential model, a game tree is utilized to demonstrate player actions, preferences, and outcomes. The weak sequential equilibrium is calculated using Gambit[1] and shows optimality in player payoffs to exist when the seller's and realtor's strategies align according to the current market. For the simultaneous model, expected payoff functions for each of the three players are constructed. PlatEMO[2], a MATLAB extension, is used to simultaneously maximize the players' functions using multi-objective optimization evolutionary algorithms. The Pareto-optimal front is found, consisting of all non-dominated solutions in the objective space. Similar to the sequential model, optimal outcomes exist when seller and realtor strategies align. Findings from both models suggest a self-interested negotiating third party is largely unnecessary and only has negative impact on player payoffs.

Contents

List of Figures

Chapter 1

Introduction

1.1 *"What's in a Game?"*

Game theory is the study of mathematical models of strategic interaction among rational decision-makers [7]. It has also been described as the mathematics of conflicts of interest when trying to make optimal choices [8]. Essentially, it provides a way to mathematically model optimal outcomes based on the players' preferences. These outcomes are dependent on the mutual actions of the players.

Applications include scenarios as simple as a group of friends trying to determine which movie to go see, to extremes such as two countries entering a trade negotiation. In these scenarios, each player has a vested interest in the outcome that is very likely conflicting with another player's interest. Arriving at an outcome that is optimal, given the players' conflicting interests, requires the use of a game theoretic model. Gillman and Housman [9] formally define a game to consist of the following:

Definition 1 A *game* consists of the following

1) A set $N = \{1, 2, ..., n\}$ of at least 2 players

2) A set O of possible outcomes that can occur when the game is played

3) Rules that state how the game is to be played

4) Defined preferences among the players involved in the game.

To build a game theoretic model, these four items must be identifiable. For example, two friends, John and Mike, may be trying to decide what movie to go see. There are three movies playing at their local theater: Movie A, Movie B, and Movie C. John would rather see Movie A which is a sequel to his favorite franchise and refuses to see Movie C due to his hatred of horror films. Mike, on the other hand, would rather see Movie C since he has already seen the other two movies playing, and Movie A is too long, given the amount of free time he has. They definitely don't want to see any of the movies playing by themselves, so they must decide amongst themselves which movie they are going see together. In this simple scenario we can identify the following:

1) **Who are the players?** In this scenario, the set of players is the 2 friends, John and Mike.

2) **What are the possible outcomes?** The set of possible outcomes is the set of all possible movies {Movie A, Movie B, Movie C} that are playing at the theater.

3) **What are the rules that govern the players' actions?** The players must work cooperatively in order to determine which movie they are going to see together. Seeing separate movies by themselves is not an option.

4) **What preferences do the players have among the outcomes?** John prefers Movie A $>$ Movie B $>$ Movie C, due to Movie A being a sequel to his favorite franchise and his hatred of horror films. Mike prefers Movie C

$>$ Movie B $>$ Movie A, due to having already seen Movie A and Movie B and his limited available free time.

The above scenario models a *cooperative game*[1] among two players with *complete information*[2]. Cooperative games allow for players to openly communicate strategies and preferences among the outcomes with each other. This awareness of the the opposing player's strategies and preferences grants each player complete information of the type of game being played. More generally, a variety of games can be constructed depending on factors such as how many players exist, whether or not there is cooperative play, if the game consists of complete versus incomplete information, if the game is being played singularly or iteratively, etc. One of the most common types of games is a mixed-strategy game, in which a player assigns a probability distribution to their available actions based on their belief of the other player's (or players') actions. Optimality exists in a mixture of strategy choices, based on a defined probability, for at least one of the players. This is a common type of outcome in game theory since pure strategy solutions are rarely seen in real-world interactions.

1.2 Buyer-Seller Games

A mixed-strategy game could also be used to describe buyer-seller interactions. Buyer-seller games abound, using various architectures and applications [9, 10, 11, 12]. These games present an interaction usually between two players, one being the seller and the other the buyer. Typically, one player (the seller) has an item which they are willing to sell at a value greater than or equal to its loss. The

[1] Cooperative games require players to arrive at a binding agreement regarding their actions [9].

[2] A player has complete information if they are aware of the rules of the game, the possible outcomes, and all preferences held by every player [9].

other player (the buyer) has a desire to purchase said item, but only at a price which they are profiting, either monetarily or otherwise. Other factors, outside of monetary gain, could influence player preferences as well. A scenario might exist where a seller has multiple buyers looking to purchase the same item. In this case, one buyer may be willing to pay more than his valuation of the item out of fear that the seller will get a better offer from another player. Alternatively, time may be a concern of the seller in that they are willing to take a reduced offer to meet a specific deadline. Depending on the nature of the scenario, all information is not always communicated between buyer and seller, indicating a lack of complete or *perfect information*[1] in the game.

Bergemann and Heumann [10] pose an example of such a game involving a seller who is privately informed of the value (v) of an item they are looking to sell. A buyer believes the true value of this item lies somewhere on a uniformly distributed interval $[x, y]$ where $0 < x < y$. Additionally, the item in question is worth $\frac{3}{2}v$ to the buyer. If the buyer proposes a price (p), the seller will either accept if $p \geq v$ or reject if $p < v$. If the seller accepts, they will get a payoff of $p - v$, and the buyer will get a payoff of $\frac{3}{2}v - p$. If the seller rejects the buyer's offer, the seller will get a payoff of v, since they are retaining the value of the item, and the buyer will get a payoff of 0. By applying a game theoretic model, an optimal offer for the buyer can be found in order to provide them with the best possible payoff, with regards to the seller's preferences [10].

As shown in the work by Bergemann and Heumann [10], interactions between the seller and buyer may either occur simultaneously or sequentially depending the on the scenario being studied. However, to illustrate the element of negotiation, buyer-seller games are more often played sequentially. This is especially apparent when applying buyer-seller games to the real estate market. Typically,

[1] Players choose their actions sequentially and know all actions taken previously [9].

buyers and sellers negotiate using a series of offers and counter-offers, responding sequentially to the previous player's action, until a mutually agreed upon resolution is achieved.

In cases of a buyer-seller interaction with missing information, the buyer and/or seller may be unaware of the other's true valuation or strategy. This lack of information between parties forces players to make decisions based off of how they believe their opponent(s) to have acted prior to their *information set*[1]; however, a negotiator could be added to the game to whom information is made freely available from both parties. This negotiator would also have a vested interest in the outcome, as they would be looking to maximize a commission or profit over time. In the real world, buyer-seller interactions are often mediated by a third party with a vested interest. Such is the case with consignment shops, online purchasing (i.e. eBay, Etsy, Amazon, etc.), and the real estate market.

1.3 The Real Estate Market

In the real estate market, sellers and buyers rarely interact directly with each other. Instead, real estate agents are typically employed by both parties. While the sale of a home can occur without the use of a negotiating party, a realtor usually acts on the behalf of the seller or buyer, mediating any buyer-seller interaction. Often the seller and buyer will have their own agents acting on their behalf, but it is possible for a realtor to act as a dual agent between both parties. In this case they would receive a double commission. This agent is tasked with pricing the home for sale, marketing the home, negotiating the sale with the buyer or buyer's agent, and facilitating the sale through closing. These services are all covered in the realtor's commission. Additionally, these agents promise a likelihood

[1]An information set is a set that, for a particular player, establishes all the possible moves that could have taken place in the game so far, given what that player has observed [13]

in increased sales price with the employment of their services. While it can be assumed that the realtor would have the client's best interest at heart, the reality is that the realtor views their client as a means off of which they can capitalize. This consideration of a third-party interest moves the two-player buyer-seller game to a three-player buyer-seller-negotiator game containing both imperfect and incomplete information, in which all three players act in self-interest.

Scenarios involving incomplete and imperfect information frequently occur. For instance, a seller who is motivated might not want this communicated to a potential buyer in effort to maximize their payoff. Additionally, a buyer may be unaware of other offers received or if there is a reserve price when determining how to bid for a property. Both of these scenarios would require an assignment of a probability distribution either across previous actions taken by the other player or across all possible game models.

Cao discusses the realtor as a third-party interest [14]. While a realtor may be hired to protect the seller's interest, they add another dimension to the game as they enter with their own interest of personal gain. Cao identifies two self-promoting strategies for the realtor. In the first, the realtor seeks to extend market time exposure of the property in order to get the best possible offer, maximizing their commission. The realtor might communicate interest in maximizing the seller's profit, but this is likely to be only to the degree in which it directly affects their commission. In the second strategy, Cao states that the realtor seeks a quick turnover, even at the risk of a lower commission. This implies they may undercut the seller's potential profit in order to maximize their own profits over time, allowing them to focus on other available listings and potential commissions. This calls to attention to the fact that seller and realtor priorities may not always align [14].

A typical commission rate for an agent acting on behalf of both parties is between 5% and 6%, depending on the type of home to be listed and the type of existing market [15]. However, this range may fluctuate between states and agencies. This rate may also be negotiated between the agent and the client. If the seller and buyer have separate agents, this commission is split evenly between the two negotiating parties. Upon closing, the seller is usually responsible to pay the full commission cost [15].

When considering player strategies in negotiation, Cao suggests the seller maintains the upper hand by communicating a rejection threshold to the buyer [14]. By announcing a rejection threshold, this informs and places an expectation on the buyer, essentially eliminating consideration of offers below a particular price point. Cao goes on to state that it is in the buyer's best interest not to inquire concerning rejected offers but rather to act according to their own optimal strategy[14]. This is considering a game only between the two parties, where all negotiations are handled without the implementation of a third party. However, the same could be easily surmised when considering interactions between the buyer and seller's agent.

1.4 Context and Overview

When analyzing buyer-seller interactions using game theory, a negotiating third-party interest is rarely considered. Any negotiations are typically conducted by the buyer and seller themselves. This study looks at the contributions a vested third-party negotiator brings to a buyer-seller game and how they influence player payoffs. Optimal strategies for the all players will be identified to determine at what point equilibria can be is achieved.

Although it is typical for the buyer and seller to have separate realtors working on their behalf, the seller's realtor is considered the only negotiating party for the purpose of this study. This negotiating third party is assumed to have perfect and complete information from both parties while remaining self-interested. This introduces a different approach to typical buyer-seller games, in which the two parties may be unaware of the other's motivations.

Buyer-seller models introduced in this paper explicitly focus on player interactions in the real estate market. Two models, an "extensive game" and a "Bayesian game", involving a third-party negotiating interest in a buyer-seller game are examined in which both incomplete and imperfect information are present. Incomplete information exists in the "extensive game" model, and imperfect information exists in the "Bayesian game" model.

1.4.1 Extensive Game Theoretic Model

The "extensive game" model implements a 50/50 chance between a buyer's and seller's market. This model deals with a single buyer, seller, and realtor and focuses on outcomes involving alignment and misalignment between the seller's and realtor's strategies and the impact this has on the players' payoffs. As buyer-seller interactions tend to be sequential, this model allows for visual representation of the sequential play and accounts for player *belief systems*[1] based on previous strategies. A game tree is constructed, showing all player *behavior strategies*[2], belief systems, and outcomes. All *weak sequential equilibria*, solutions that are *sequentially rational*[3] and demonstrate *consistency of beliefs*[4] among the players, are identified.

[1]A belief systems is a function that assigns a probability distribution over histories in each information set not assigned to chance [9].

[2]A behavior strategy is a function which assigns to each of the player's information sets a probability distribution over possible actions [9]

[3]A sequentially rational solution requires all player strategies to be a best response at each information set to which either the player or chance is assigned [9]

[4]Consistency of beliefs is held when a player's belief system matches that of the previously acting player's strategy profile [9].

1.4.2 Bayesian Model

The “Bayesian game” model deals with a single buyer, seller, and realtor but considers the existence of different types of the negotiating party. Two types of realtor interest are considered: a realtor who desires to maximize their commission and a realtor looking for a quick sale, as these are the two dominating interests of a negotiating third party in the real estate market. Variables such as the agreed upon list price between the seller and negotiator, commission rate selection by the negotiator, and the buyer’s offer price are analyzed to determine where optimal payoffs occur. Expected payoff functions for each player are constructed using these variables. Parallel processing of the player’s expected payoffs using a *non-dominated*[1] sorting genetic algorithm is then utilized to find the *Pareto-optimal front*[2] of the simultaneously maximized player functions using PlatEMO [2] in MATLAB [16]. The Pareto-optimal front is then identified to determine the set of non-dominated solutions that exist for each of the players and how payoffs are influenced by the realtor.

[1]A solution is called non-dominated if none of the objective functions can be improved in value without degrading some of the other objective values.

[2]A Pareto-optimal front is a collection of all non-dominated points within the objective space

Chapter 2

Model Architecture

2.1 Case I: Extensive Game

In sequential games, consideration must be given to the player's knowledge of previous actions at each *non-terminal history*[1] as well as their knowledge of the type of game they are playing (i.e. the rules, outcomes, and preferences). When a player possesses knowledge of all previous actions, the player is said to have perfect information, and when the player is aware of all rules outcomes, and player preferences, the player is said to have complete information. Games in which players take action sequentially with incomplete and/or imperfect information are known as *extensive games*. Gillman and Housman [9] formally define extensive games as consisting of the following:

Definition 2 An *extensive game* consists of the following

1) A set $N = \{1, 2, ..., n\}$ of at least two players

[1]A non-terminal history marks a subsequence of play that is made along the way before reaching the end of the game [9].

2) A set O of *terminal histories*[1]

3) Information sets partitioning all non-terminal histories such that each non-terminal history within an information set has the same set of possible actions following it.

4) A player or chance is assigned to each information set.

5) For each information set assigned to chance, there is a probability distribution that chance uses to select its action.

6) Starting with the *empty history*[2], if a non-terminal history is reached, the assigned player, or chance, selects an action to append to the current history.

7) Utility functions $u_i : O \longrightarrow \mathbb{R}$ that specify preferences among terminal histories for each player $i \in N$

Sequential games, like the extensive model presented in this text, are typically represented using a *game tree*[3]. The tree's root node, also called the empty history, indicates the starting point of the game. Edges branching from nodes represent actions either assigned to players or chance. Nodes within the game are non-terminal histories and represent player sequences within the game. Terminal nodes are labeled with outcomes/payoffs for their particular sequence of actions [9].

[1] A terminal history marks the ending sequence of a sequential game.

[2] An empty history marks the starting sequence of a sequential game.

[3] A game tree provides a visual representation of a sequential game and displays all ways that a game could be played.

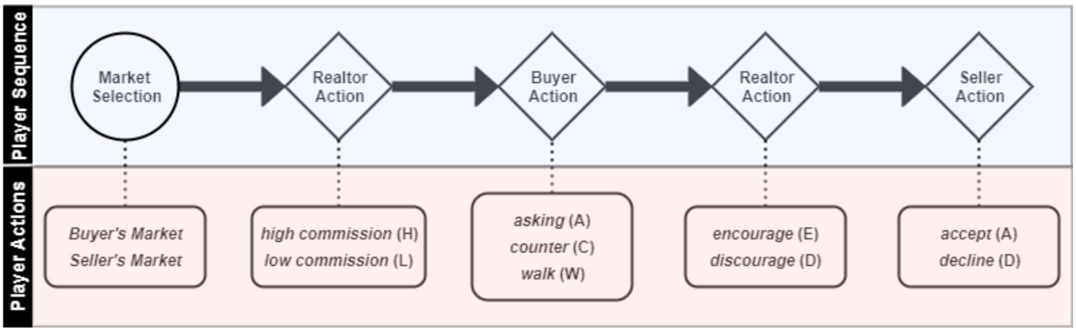

Figure 2.1: Sequence of play for all players in the extensive model along with their available actions at each turn (abbreviations correspond to actions in Figure 2.2)

2.1.1 Building the Game Tree

We construct a three-player extensive model involving a single seller, buyer, and realtor; all of whom are looking to maximize their payoff. Chance is implemented as the root node with a 50/50 probability of either being a buyer's market or a seller's market. A buyer's market suggests an oversaturation of homes on the market, allowing the buyer a greater likelihood of acceptance of a reduced offer. A seller's market suggests a dearth of homes on the market, allowing the seller a greater likelihood of achieving an increased sales price. As with any modeling process, several assumptions will be made in regards to the players and their actions.

> **Assumption 1.** The type of market is directly related to the seller's motivation (i.e. A buyer's market indicates the seller is motivated while a seller's market indicates the seller is indifferent).

Following declaration of the market, all players are aware of the current market and can choose to act accordingly. The realtor takes action first, choosing either to list the property high in effort to maximize their commission (H), extending its market time, or list the property low (L), reducing its market time.

Assumption 2. The realtor action (H) implies that the property is listed at the high end of *fair market*[1]; alternatively, (L) implies the property is is listed at the low end of fair market.

Assumption 3. The realtor action (H) implies an increase in time on the market for a potentially larger purchase price, while (L) is a decrease in time on the market for a potentially smaller purchase price.

The buyer's action follows in which they must decide to make an offer for the property at the asking price (A), submit a counter offer reasonable for the current market (C), or refuse to even bid the property and walk away (W). If the buyer chooses to submit a counter offer, the action sequence then goes to the realtor; otherwise, the game ends.

Assumption 4. The buyer action (C) implies a reduced offer, typical for the current market, is made.

At this point in the game, the buyer, although aware of the current market, has imperfect information regarding the realtor's previous action. They are unaware if the realtor has listed the property at an inflated price to increase their commission (H) or if the listing price has been reduced to facilitate a quick sale (L). Thus, a belief system must be implemented, due to the information set consisting of two possible node locations within the game.

The realtor's second action sequence requires them to either encourage the seller to accept the counter offer (E), indicating the counter offer still affords a reasonable commission, or discourage acceptance of the offer (D), indicating the contrary.

[1]For this study, fair market will be defined as the interval $[x, y]$ on which market value exists.

Assumption 5. In regards to the realtor action (H), the follow-up action (E) indicates the realtor is still within their desired commission range, while (D) indicates the realtor's commission would significantly drop.

Assumption 6. In regards to the realtor action (L), the follow-up action (E) indicates the realtor values time saved over a reduced commission, while (D) indicates the time saved is not worth the reduced commission.

The seller takes actions last, and only if a counter offer is presented. In such a scenario, the seller must either accept the reduced offer (A) or reject the offer (R) on the basis of their motivation. Imperfect information exists at this player sequence as well. For each market, two separate information sets exist for the seller. While the seller is aware of the realtor's second action (E) or (D), he is unaware of the realtor's initial action (H) or (L); therefore, both terminal histories following (E) make up one information set and both terminal histories following (D) make up the other. Similar to the buyer, the seller must implement a belief system based on how he believes the realtor is acting in the game - looking to maximize his commission or looking for a quick turnover.

Players are concerned with a variety of factors, depending on where their action point is within the game. For instance, in a buyer's market where there is an oversaturation of houses on the market, the seller would place primary importance on reducing market time and secondary importance on achieving their valuation. Alternatively, in a seller's market where demand exceeds supply, the seller is likely to be indifferent about time, placing all importance solely on their return value.

Assumption 7. A motivated seller places most importance on time and secondary importance on purchase price; conversely, an indifferent seller place most importance on purchase price and are unconcerned with time.

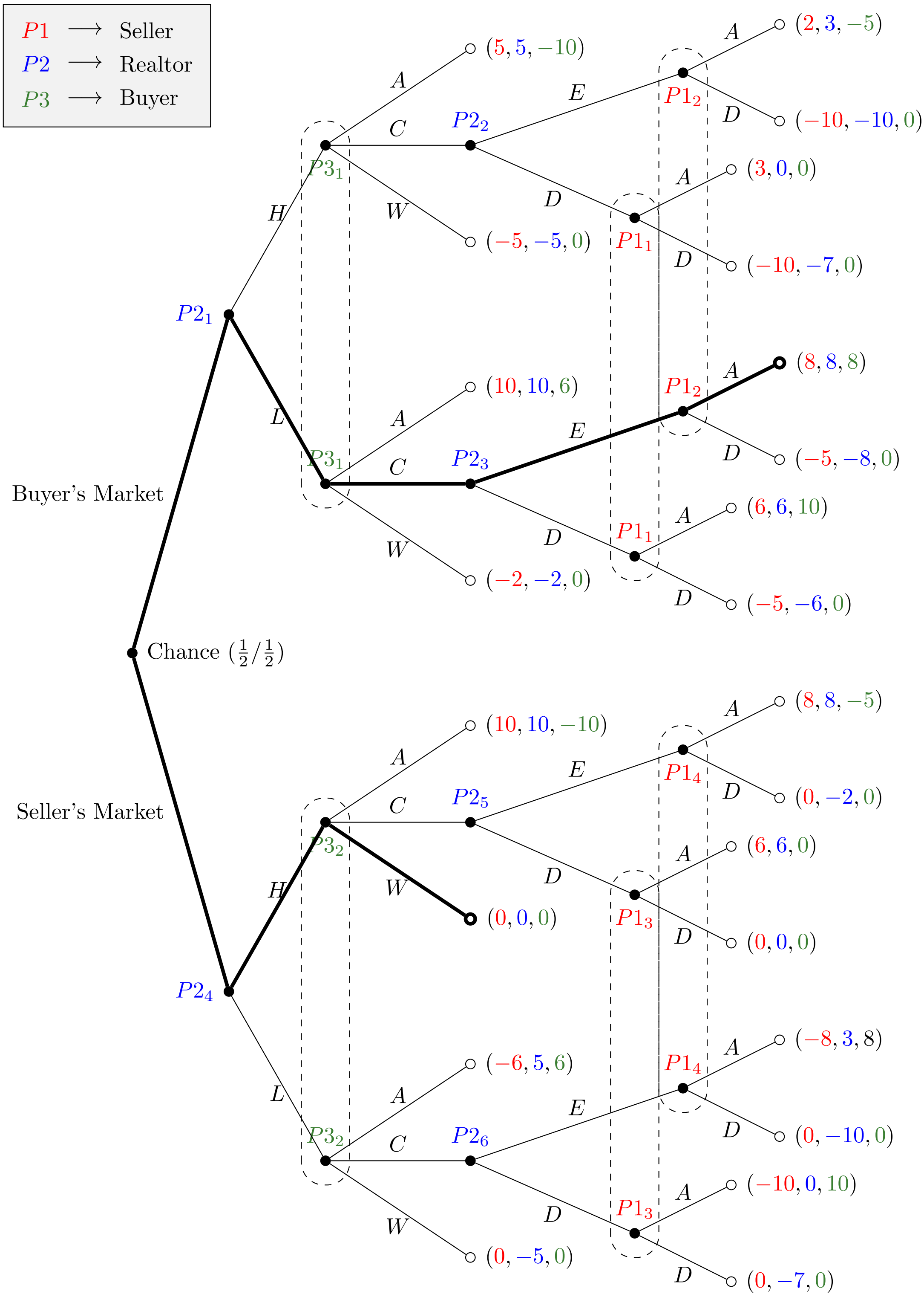

Figure 2.2: Extensive model game tree showing sequence of play, player actions (e.g. H, L, A,...), information sets (e.g. $P1_1$,..., $P2_1$,...,$P3_1$,...), and payoffs (#,#,#).

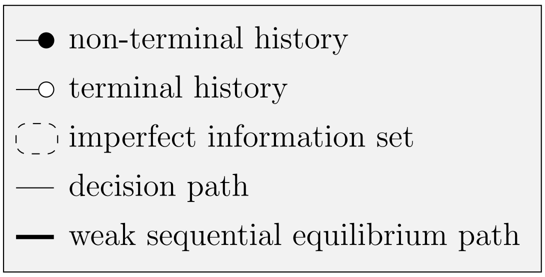

Figure 2.3: Technical legend for Figure 2.2

A buyer is only ever concerned with receiving the best possible deal, regardless of the type of market they are in.

Assumption 8. The buyer's valuation is synonymous with market value.

Assumption 9. Market value is the midpoint on the fair market interval.

The realtor, however, must have a variety of considerations, depending on where they are looking to make a decision. At information set $P2_1$, the realtor must decide to either go for a higher (H) or lower (L) commission, corresponding to an extended market time or a quick sale, respectively. Since this information set is in a buyer's market, to act in the seller's best interest would mean that the realtor would need to consider time to be of most importance, therefore choosing (L). Choosing this option, the realtor's payoff would have three considerations: time saved, commission received, and the possibility of the client being retained for future services. If action (H) was selected by the realtor at information set $P2_1$, time would be of no concern and the payoff would only have two considerations, the commission received as well as the possible loss of a future/current client. At information set $P2_4$ the realtor's decision exists in a seller's market. To choose (H) would place primary importance on obtaining a larger commission, which is directly related to the sales price. Seeing how the seller is indifferent about time and is only concerned with obtaining the largest return value as possible, the realtor's payoff takes into consideration the commission received as well as the possibility of the client being retained for future services. However, to choose (L), would again result in three considerations to the realtor's payoff: time saved, commission received, and the possible loss of a future/current client.

Payoffs in Figure 2.2 are calculated using previously stated considerations as if players had perfect information as to how the game was played. Player payoffs exist on the interval $[-10, 10]$, where -10 indicates the worst possible outcome for a player, 0 indicates the player gains/loses nothing in regards to the considerations associated with that outcome, and 10 indicates the best possible outcome for a player. All other payoff values are a sum of gains and loses in regards to considerations they hold at that particular outcome.

For example, at the terminal history "Buyer's Market, H, A", a buyer's market is selected, the realtor chooses to list the property high for a larger commission, and the buyer puts in an offer at the asking price. The assigned player payoffs here are $(5, 5, -10)$. Here the seller gets a reduced payoff of 5, due to market time being extended, per the realtor's actions, while they are motivated to sell. It is important to remember that in a buyer's market the seller values reduced market time more than an increased sales price. Even with this being the case, they attain a payoff greater than 0 since they are obtaining a sale price significantly over market value, although still within fair market. The realtor also receives a reduced payoff of 5, due to their loss of potentially retaining a future client because of misalignment in strategy choice. However, a payoff greater than 0 is assigned since they are obtaining a significantly larger commission. The buyer receives the worst possible payoff, -10, since they are paying significantly over market value in a market where the seller is motivated.

2.1.2 Calculating the Weak Sequential Equilibrium

To calculate the weak sequential equilibrium, the game tree form Figure 2.2 is constructed in Gambit[1] [1]. Once the game is initiated in the GUI, sequentially rational behavior identified and consistency of beliefs are implemented. The output defines the weak sequential equilibrium for the game and calculates the players' expected payoffs.

Each best response action for all players at every non-terminal history are identified and the edge labeled with their appropriate action is highlighted. These highlighted actions demonstrate sequential rational behavior for the player with whom the action is associated. For instance, when looking at any one of the seller's non-terminal histories in the buyer's market *subgame*[2] (see Figure 2.2), it is apparent that they will always get a better payoff by selecting (A) for acceptance of the offer rather than selecting (D) to decline the offer. This makes sense since in a buyer's market it is assumed that the seller is motivated and willing to take a loss in monetary gain for the sake of time. Therefore, we could say that, within the buyer's market subgame, the seller's strategy to decline is dominated by their strategy to accept.

A belief system is incorporated at information sets $P3_1$, $P3_2$, $P1_1$, $P1_2$, $P1_3$, and $P1_4$. Each belief system is assigned based on how the deciding player believes the previous player to have acted based on the information known. Consistency of beliefs is demonstrated at each information set based on how the acting player believes the other to have acted. For example, at information set $P3_2$, the buyer must decide whether to take the property as asking, submit a counter offer, or walk away. The buyer is aware that a seller's market exists, but they are unaware

[1] Gambit is an open-source collection of tools for doing computation in game theory.

[2] A subgame $G(h)$ of the sequential game G, beginning at the non-terminal history h, consists of the players in the game, any terminal histories for which h is its initial party, and the player function and preferences inherited from the full game. The game G is a subgame of itself, and there is a proper subgame for each non-terminal history of the game [9].

if the property was listed high, to increase the realtor's commission, or low, for a quick sale. While they can achieve a payoff of 6 by taking the property at asking if a low list price was submitted, they can receive a payoff of -10 if a high list price was submitted. From their knowledge of the seller's market, it is in their best interest to walk away due to their belief that the realtor would list high in such a market. This assigns them a payoff of 0 rather than -10 or, potentially, -5. Seeing as the buyer's belief system matches that of the realtor's strategy at $P2_4$, consistency of beliefs has been established.

The weak sequential equilibria is stated using an *assessment* of the players' strategy profiles (s) as well as their beliefs (β). Strategy profiles reflect sequentially ration behavior for the players at each of their information sets, while beliefs reflect a consistency of beliefs as to how each player believes the opposing player(s), for which they have imperfect information, to have acted. The following is the assessment for the weak sequential equilibrium derived from this game model.

$$(s, \beta) = ((s_{Seller}, s_{Realtor}, s_{Buyer}), (\beta_{Seller}, \beta_{Buyer}))$$

where

$$s_{Seller} = (s(P1_1), s(P1_2), s(P1_3), s(P1_4)) = (A, A, A, A)$$

$$s_{Realtor} = (s(P2_1), s(P2_2), s(P2_3), s(P2_4), s(P2_5), s(P2_6)) = (L, E, E, H, E, E)$$

$$s_{Buyer} = (s(P3_1), s(P3_2)) = (C, W)$$

$$\beta_{Seller} = (\beta(P1_1), \beta(P1_2), \beta(P1_3), \beta(P1_4)) = (L, L, H, H)$$

$$\beta_{Buyer} = (\beta(P3_1), \beta(P3_2)) = (L, H)$$

As defined by the assessment above, the expected player payoffs based on the weak sequential equilibrium is

$$\frac{1}{2}(A; L, E; C) + \frac{1}{2}(A; H, E; W) = \frac{1}{2}(8, 8, 8) + \frac{1}{2}(0, 0, 0) = (4, 4, 4). \tag{2.1}$$

2.1.3 Interpreting the Equilibrium

This interprets such that, if in a buyer's market, the realtor should focus on listing the property for quick sale. The buyer, aware of the current market, should counter under the assumption that the seller is motivated. Under the assumption that a reasonable counter has been made within the range of fair market, the realtor should, in turn, encourage the seller to accept the offer as this would allow him to received a reduced commission but as quickly as possible. The seller, who is motivated under the current market, should accept the reduced offer.

If in a seller's market, the realtor should extend market exposure by listing the the property at the higher end of fair market, in order to receive the highest possible commission. Accordingly, a rational buyer, strictly looking at monetary gain, should walk away from the offer. In such a scenario the seller would not even get a chance to act.

It isn't trivial to point out that the realtor, as a negotiating third party with vested interest, maximizes their profit when aligning their strategy with that of the seller. This means in a buyer's market, the realtor should place higher priority reducing market time rather than extending market exposure to possibly attain a higher commission. In a seller's market, where the seller is indifferent about selling unless presented with a substantial offer, the realtor should be unconcerned with time and extend market exposure in maximize their commission. As can be seen in Figure 2.2, when the realtor and seller strategies align, payoffs dominate outcomes with misalignment.

2.2 Case II: Bayesian Game

Bayesian games look not only at the number of players but also at potential types of each player as well. This introduces incomplete information into the game, as players are unaware of the type of players they are facing and must incorporate a probability distribution over all possible types. This provides a more realistic approach to buyer-seller behavior as the market isn't always indicative of how a player values a property. In the extensive game, it was assumed that a buyer's market was indicative of a motivated seller while a seller's market was indicative of an indifferent seller. Since players were aware of the current market, they could rightfully assume the actions of their opponent(s), this of course being if the realtor aligned their strategy with that of the seller. Bayesian games make no such assumptions. Therefore, this removes the need for consideration of the type of market all together. Rather, player values exist on intervals, where conditionals dictate player payoffs. Gillman and Housman [9] formally define Bayesian games as consisting of the following:

Definition 3 A *Bayesian game* consists of the following

1) A set $N = \{1, 2, ..., n\}$ of at least 2 players.
2) A set T_i of types for each player i such that $T = T_1 \times T_2 \times ... \times T_n$ is the set of type profiles known as the type space.
3) A set A_i of actions available to players i for each $i \in N$.
4) A set of outcomes $O = A = A_1 \times A_2 \times ... \times A_n$ that occur when typed players choose their actions according to their type profiles
5) Belief functions ϕ_i where $\phi_i(t_{-i}|t_i)$ is the probability that the other player types come from the type profile t_{-i} given that player i has type

t_i. The function ϕ_i is a probability distribution on the truncated type space $T_{-i} = T_1 \times T_2 \times T_{i-1} \times T_{i+1} \times ... \times T_n$. (The notation $t_{-i} \in T_{-i}$ indicates a type profile with t_i removed.)

6) A rule stating that players simultaneously choose actions after privately learning their own types.

7) Utility functions u_i where $u_i(a|t_i)$ is the cardinal utility player i of type t_i ascribes to outcome a, for each player i.

While we are neglecting to consider the type of market, several assumptions will still need to be considered in determining exactly what market value value is and the player's perception of that value. The following assumptions are retained from the extensive model:

Assumption 1. The buyer's valuation is synonymous with market value.

Assumption 2. Market value is the midpoint on the fair market interval.

In addition, the following assumptions will be made in regards to the market:

Assumption 3: Market value accounts for location, school district, recreational facilities, and other such amenities.

Assumption 4: The seller values the property at precisely market value.

Although real estate, as much as any other type of good to be sold, can elicit emotional and irrational behavior from both buyer or seller, it is important to remember that game theory only considers rational decision making. The best way this can be represented is by calculating players payoffs solely through gains or losses of monetary value.

Assumption 5: The seller and buyer are strictly motivated by monetary gain.

Using what we know about about the market and how players perceive it, we can define the expected payoff utilities for each of the players.

2.2.1 Defining the Players' Utilities

When defining the player utility functions, consideration must be given to when an offer would be accepted or otherwise. To maintain rationality in this decision, a reserve price (p) will be introduced to the the model. This strategy, while selected by the seller, is strategically influenced by the realtor in attempt to achieve their preferred outcome: a maximized commission as a result of a maximized sale price, a reduced commission as a result of a quick sale. Additionally, this allows the seller to, potentially, maximize their sale price while also rejecting all other offers below that threshold. It is important to note that this value can be above, below, or directly at market value, based on the the seller's and realtor's motivation. The seller still values the property at market value. In the event that the seller sets a reserve price below market value (m) this would indicate the seller, motivated by factors not accounted for in this model, is willing to take a negative payoff. This must be understood in order for rational play to be considered consistent.

> **Assumption 6**: The seller's reserve price p of the property is a privately known, realtor-influenced, lowest acceptable offer price.

2.2.1.1 Seller's Utility Function

$$E_S = Pr\{v \geq p\} * E\left[\frac{1}{2}(v - m - c_L) + \frac{1}{2}(v - m - c_H)\right] + Pr\{v < p\} * E[m] \quad (2.2)$$

The seller's utility function is defined by the buyer's offer (v), the current market value (m), and whether or not they are dealing with a realtor seeking a high commission (c_H) or low commission (c_L). This utility function is inclusive of the seller's payoff under acceptance and rejection of the buyer's offer. A probability of acceptance ($Pr\{v \geq p\}$) is assigned for conditions under which the buyer's offer is at least equal to or greater than the seller's reserve price. Under this condition, the expected payoff (E) is buyer's offer minus the minus the market value and the realtor's commission. A sum of the payoffs, given the likelihood of the two types of realtors, is included in the utility. In the event that the $v \leq m$ the seller would receive a negative payoff under the accepted conditional. The buyer's offer must be greater than market value by an amount equal to that of the realtor's commission in order for the seller to simply break even. In the event that the buyer's offer is less than the seller's reserve price ($Pr\{v < p\}$), the seller would receive an expected payoff of the market value, since they retain the value of their property.

2.2.1.2 Buyer's Utility Function

$$E_B = Pr\{v \geq p\} * E[(m - v)] + Pr\{v < p\} * E[0] \tag{2.3}$$

The buyer's strategy is defined by the variable v. Similar to the seller's reserve price, the buyer's offer may be greater than, less than, or equal to market value. This strategy is representative of how motivated the buyer is to purchase the property in question. Since each payoff is defined by monetary gain to the player, the expected payoff to the buyer (E), under the condition that the offer is accepted ($Pr\{v \geq p\}$), is calculated by finding the difference between market value and the buyer's offer. A negative payoff to the buyer would occur if their strategy causes them to exceeds market value. While this may seem irrational on the part of the buyer, their motivation to exceed market value might indicate a seller's market with a higher reserve price. A scenario in which the buyer always values the property at market value but uses v as a strategy to achieve the best deal possible could be explored in future study but will not be analyzed in this model. If the buyer's offer fails to at least meet seller's reserve price ($Pr\{v < p\}$), they would receive a payoff of 0, since there is no monetary gain or loss.

2.2.1.3 Realtor's Utility Function

$$E_R = Pr\{v \geq p\} * E\left[\frac{1}{2}c_L(v) + \frac{1}{2}c_H(v)\right] + Pr\{v < p\} * E[0] \tag{2.4}$$

The realtor's strategy is in selection of their commission rate. A realtor looking to maximize their commission would would select c_H while a realtor more concerned with making a quick sale would likely select c_L. This presents the realtor with two different types in their type space. A belief function, ϕ_i, is applied as a probability distribution over the type space. In the case of the realtor's utility function a 50/50 probability has been applied to either of the realtor's types. Their expected payoff, under the condition that the buyer's offer is accepted ($Pr\{v \geq p\}$), is the sum of their selected commission rate multiplied by the buyer's offer over the type space. If the buyer fails to submit an acceptable offer ($Pr\{v < p\}$), the realtor would receive an expected payoff of 0, since they gain nothing but likely retain their client for future offers.

Player utilities are composed as the sum of two separate expected payoffs under opposing probability conditions - ($Pr\{v \geq p\}$) and ($Pr\{v < p\}$). Since v and p are independent of each other, as neither player is aware of the opposing player's strategy/offer before the onset, a probability of $\frac{1}{2}$ has been assigned to each. The player utilities can be further reduced to the following:

$$E_S = \frac{1}{2}(v - \frac{m_L + m_H}{2} - \frac{c_L(v) + c_H(v)}{2}) + \frac{1}{2}(\frac{m_L + m_H}{2}) \tag{2.5}$$

$$E_B = \frac{1}{2}(\frac{m_L + m_H}{2} - v) + \frac{1}{2}(0) \tag{2.6}$$

$$E_R = \frac{1}{2}(\frac{c_L(v) + c_H(v)}{2}) + \frac{1}{2}(0) \tag{2.7}$$

where

$$p, v \in [m_L, m_H], \qquad c_L, c_H \in [0.05, 0.06]$$

The realtor's commission, c_L and c_H, exists on separate intervals where the values of the *max* and *min* on the bounded interval are typical across various market climates. For the purpose of this study, the bounded interval for c_L will be defined as $[0.05, 0.055]$ and the bounded interval for c_H will defined as $[0.055, 0.06]$. Random selection across these intervals is uniformly distributed for both types of realtor. Since counter negotiating is not being considered in this model, the realtor's payoff is consequentially assigned according to the selected commission of the final sale price as opposed to the selected list price, as seen in the previous model.

Fair market is now defined as the interval $[m_L, m_H]$, where m_L is the lower bound of fair market and m_H is the upper bound. The seller's reserve price and the buyer's offer both exist on this interval. Also, market value, previously defined as the midpoint of fair market, has been rewritten in terms of the interval as $(m_L + m_H)/2$.

2.2.2 Simultaneously Played Games

As game theory looks to maximize payoffs for players of conflicting interest, this may happen through sequential or simultaneous play. In simultaneous games, players select their strategy independently from each other. After all players select their strategy the game is initiated and everyone plays at once. Payoffs are then assigned according to a defined utility function. Since our model has three utility functions (E_S, E_B, E_R), we maximize them simultaneously in order to achieve optimality. Multi-objective optimization is incorporated, where each player's payoff function is an objective function to be maximized. The next chapter expands on these methods to calculate optimal payoffs for each of the players using evolutionary multi-objective optimization.

Chapter 3

Methodology

3.1 Mulit-Objective Optimization

A multi-objective optimization problem (MOOP) looks at multiple functions in which the goal is either to maximize or minimize each function simultaneously. Each function is composed of variables, known as *decision variables*, and constraints that directly impact the outcome. The space in which all possible values for the decision variables exist is known as the *decision space*. Constraints on the decision variables will have an effect on the size and shape of the decision space. The number of decision variables across the objective functions determines the dimensionality of the decision space. Multi-objective optimization maps reference points within the decision space to a separate space defined by the objective functions known as objective space. Similar to the decision space, the number of objective functions determines the dimensionality of the objective space. Dimensionality between the decision and objective spaces need not be uniform. To generalize, multi-objective optimization is a mapping between an n-dimensional solution vector and an M-dimensional objective vector [4]. An illustration of this idea can be seen in Figure 3.1.

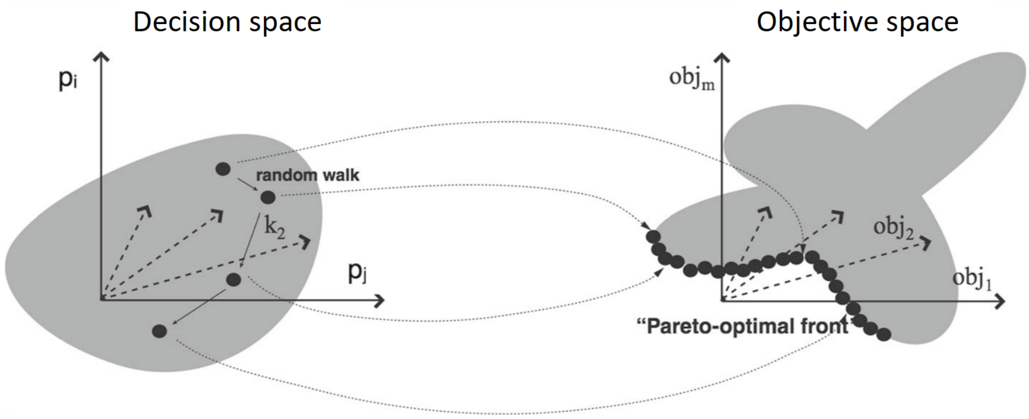

Figure 3.1: Schematic of multi-objective mapping. The performance of any given parameter set is mapped into an objective space using a ranking function which quantifies the quality of the parameters. This image has been reproduced from Bassen et al [3].

Once the objective space has been composed of the specified population of reference points from the decision space, a *Pareto-optimal solution*[1] is found. In MOOPs, there is no singular solution, but rather a set of solutions dependent on whether the objective functions are looking to be maximized or minimized. This set of possible solutions is known as the *Pareto-optimal front*. Within the Pareto-optimal front sits all possible non-dominated values that meet the conditions of the system of objective functions. The Pareto-optimal front is defined by the reference points that sit on that edge of the objective space. Inferences can be made as to what the true Pareto-optimal front looks like based on the population of reference points. The illustration above (Figure 3.1) maps a Pareto-optimal front for which the objective functions are being minimized.

A common MOOP considers decision-making during the car-buying process. A buyer might consider a variety of factors when looking to buy a car; two of which could be the cost and comfort of the vehicle. For this example, it is safe

[1]A Pareto-optimal solution is a solution where a trade-off exists between values such that no value in the solution can be increased/decreased without decreasing/increasing another [4].

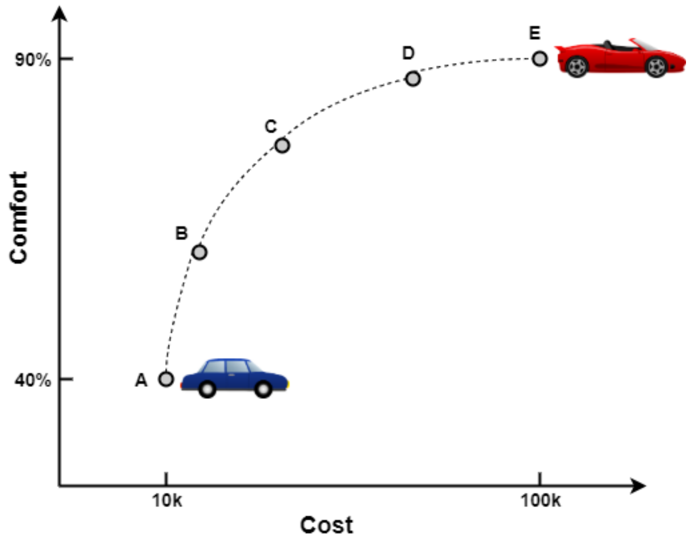

Figure 3.2: Trade-off solutions for an illustrated car-buying multi-objective problem [4].

to assume that an inexpensive car is less comfortable than an expensive one (see Figure 3.3). If the buyer's only objective is to reduce the cost as much as possible, then solution A would be the optimal choice. Alternatively, if the buyer is only concerned with maximizing the comfort of the vehicle, then solution E would be the optimal choice. In either scenario, the buyer holds a single objective which affords them a singular solution. However, if the buyer is looking to maximize the comfort while minimizing the cost, this presents an MOOP. The curved line in Figure 3.3 represents the Pareto-optimal front, and the points on that curve represent Pareto-optimal solutions. In our case, each dot represents a different vehicle the buyer might consider given their objectives. Each one of these points on the Pareto front are non-dominated, meaning there is a trade-off between cost and comfort as you move across the curve. Moving from one point to the next will increase one of the objectives while decreasing the other. Therefore, A, B, C, D, and E are all considered to be solutions to the MOOP [4].

3.1.1 Evolutionary Algorithms

Evolutionary algorithms (EA) are a computational, heuristic-based approach to solving MOOPs. The basis of how these algorithms work is rooted in biological evolution, particularly natural selection [5]. In an EA, ideal members will survive and proliferate, while unfit members will die off, failing to contribute to the gene pool of further generations. This happens through a four-stage iterative process of initialization, selection, genetic operators, and termination.

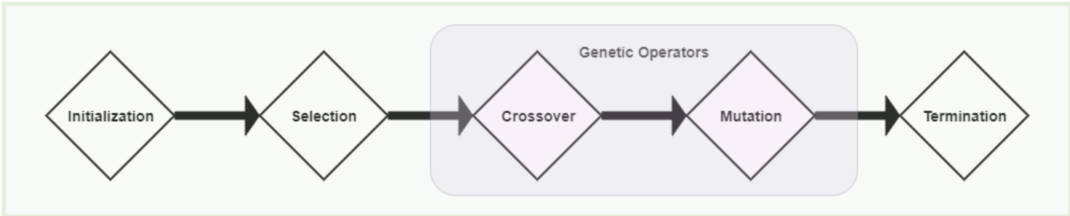

Figure 3.3: Natural selection process for evolutionary algorithms [5].

3.1.1.1 Initialization

In this first stage, a population of randomly generated members will be selected from the decision space. The size of the population will determine the number of possible solutions in the output. This could also be thought of as the number of reference points to populate the Pareto-optimal front. While population selection is random, a distribution could be applied to the selection, whether that be uniform, beta, etc [5].

3.1.1.2 Selection

During selection, the each member of the population is evaluated according to the objective functions in the MOOP. Non-dominated solutions are identified as part of the Pareto-optimal front. These members form the bound for all other dominated solutions [5].

3.1.1.3 Genetic Operators

Since the goal is to populate the Pareto-front with the entire population, genetic operators are applied to output of the selection process. In the crossover operator, the top, most well-fit, members are selected as a parent population. The size of this parent population can differ based on the EA being used. This new parent population determines the next generation of the algorithm by selecting a offspring population based on the genetic information its current members. The members of the offspring population has a mixture of the genetic qualities the parent population possesses [5].

Next, a mutation is applied across the new offspring population to ensure they no longer perfectly mirror the genetic subsets of the parent population. This prevents the optimized output from being stuck in a local extrema, failing to show true optimality in the results. Typically, the chance of the offspring receiving the mutation as well as the intensity of the mutation is determined by a probability distribution [5].

3.1.1.4 Termination

The final stage of the evaluation process occurs under one of two conditions. 1) The specified number of generations is reached. 2) The threshold of performance is reached. When either of these conditions occur, the process terminates, and the output is provided [5].

3.1.2 Evolutionary Multi-objective Optimization (EMO)

This study uses evolutionary algorithms that emphasize non-dominated solution sets, as game theory particularly looks at non-dominated outcomes. If P is to be the set of all feasible values in objective space, then P' is the subset of all non-dominated solutions, also referred to as the Pareto-optimal set. Evolutionary algorithms are able to find these non-dominated solutions efficiently across a variety of objective spaces, including those that might be discontinuous or concave [4].

A multi-objective evolutionary algorithm (MOEA) typically uses one of two methods to find a Pareto-optimal front. The first method looks to find the best non-dominated solutions for a population. Once each member of the population are mapped into the objective space, they are individually compared with one another to determine dominance. If a member is non-dominated then it is added to the subset P'. This process is continued until all members have been compared with each other, and P' is solely comprised of non-dominated solutions [4].

The second method takes a somewhat different approach to the previous method, assigning members of the population to different levels of dominance. Rather than isolating non-dominated solutions from all other members of the objective space, all members are assigned to one of potentially many fronts. The process is continued across all members of the population until no members are left without an assigned front. The number of levels is determined by the spread

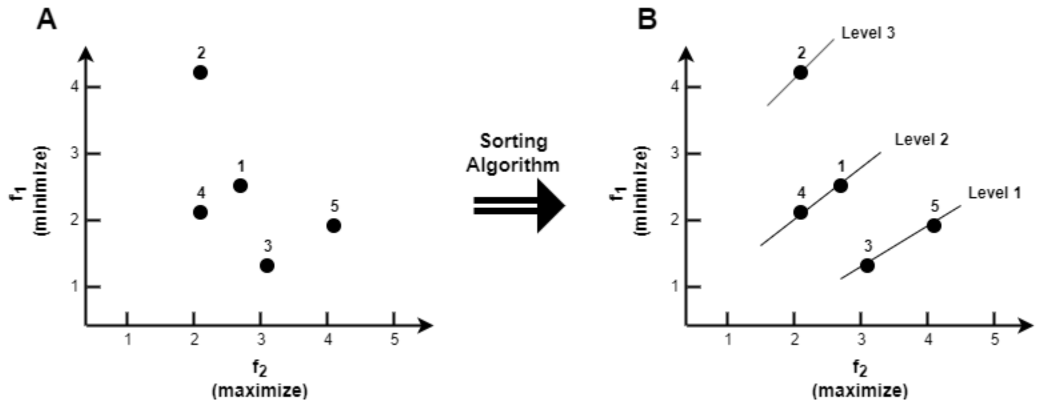

Figure 3.4: Sorting of population members into non-dominated fronts based on levels of dominance [4]

and shape of the population in objective space. Those solutions populating the Level 1 front are considered to be most dominant, while subsequent levels are less dominant (see Figure 3.4). Once a member is sorted into its appropriate non-dominated level, it is never visited again, reducing the computational complexity that is required for the first method [4].

3.1.2.1 NSGA-II

One very popular MOEA that utilizes a non-dominated sorting method is NSGA-II (non-dominated sorting genetic algorithm-II). A schematic of how this algorithm works can be seen in Figure 3.5. To start, an offspring population is generated using genetic operators on the parent population (N), such that the combined population of parent and offspring is equal to twice the size of the original parent population ($2N$) [17]. Non-dominated sorting assigns every member of the population to a front based on each member's overall dominance. Since the new generation's population needs to be the same size as the original parent population, members are selected starting with Front 1 and moving to subsequent fronts until a front must be split in order to fill the remainder of the new generation. At this point a *crowding distance*[1] metric is utilized to determine what members of that specific front are best. Crowding distance is calculated using the following formula [18]:

$$CD_{im} = \frac{f_m(x_{i+1}) - f_m(x_{i-1})}{f_m(x_{max}) - f_m(x_{min})}, i = 2, ..., (l-1) \tag{3.1}$$

In the formula above f_m is the objective function, x_i is the reference point, x_{i+1} and x_{i-1} are the two neighboring solutions, x_{max} and x_{min} are the maximum and minimum values in the population, and l is the size of the population. Boundary members on the front are assigned a crowding distance of ∞, since they don't

[1] Crowding distance calculates the average distance of a member's two neighboring solutions.

have neighboring solutions on either side. The crowding distance for all members between the boundary points is calculated using the metric given above. Fro each objective functions, the outputs of the two neighboring solutions are subtracted and then divided by the difference in the outputs of the minimum and maximum values in the population. This, in a sence, normalizes the calculated distance. The crowding distance for each member on the front is then added across all objective functions.

$$CD_i = \sum_{m=1}^{M} CD_{im} \tag{3.2}$$

Those members with the larger crowding distance are selected to be a part of the next generation. This allows for greater exploration of the objective space [18].

All remaining members of the split front and all succeeding fronts are then removed from the gene pool. This entire process comprises the construction of one new generation of the parent population. Through programs, such as PlatEMO, NSGA-II can be run for hundreds, even thousands, of generations in just a short period of time, each new generation providing a more accurate representation of the Pareto optimality.

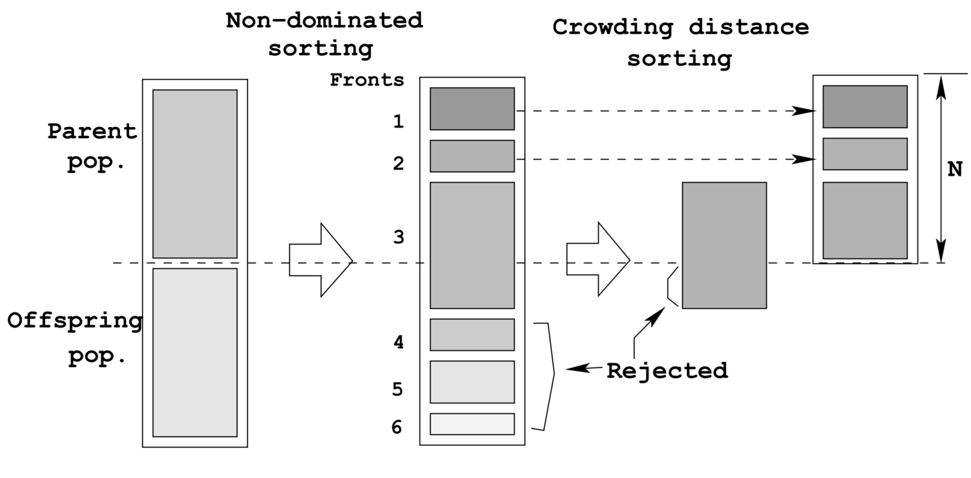

Figure 3.5: Schematic of the NSGA-II procedure. This image has been reproduced from Mittal and Deb [6].

3.2 Implementation

3.2.1 Tools

The objective functions for each of the players were coded and run in a MATLAB-based evolutionary multi-objective optimization platform extension known as PlatEMO [2]. Developed in 2017, this platform allows for users to run and test several different multi-objective problems using a variety of evolutionary algorithms such as NSGA-II, NSGA-III, SPEA2, ϵ-MOEA, etc. While other MOEA libraries do exist, PlatEMO is unique in that it offers users a GUI for ease of access and output comparison between algorithms. PlatEMO is also easily extensible, allowing users to code in and use their own MOOP and algorithms, all of which is full developed in the MATLAB language.

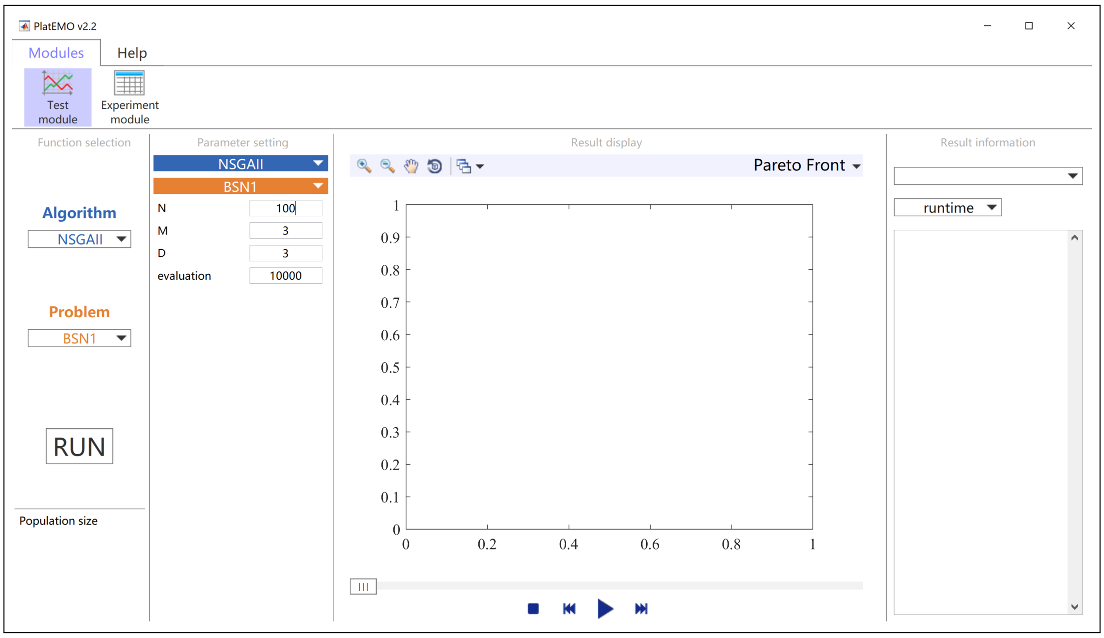

Figure 3.6: GUI for the test module of PlatEMO

Within the interface, the user defines the problem to be solve and selects the algorithm to apply. Next, the population size (N), number of objective functions in the problem (M), number of decision variables (D), and maximum number of fitness evaluations must be specified. Once all parameters are selected, the user must simply initiate by clicking the play button at the bottom of the GUI. A graphical animation can be seen as the MOOP is run through the algorithm over the specified evaluations.

3.2.2 Coding the Problem

Using the PlatEMO extension in MATLAB, a new MOOP labeled BSN1 (i.e. Buyer-Seller-Negotiator-1) was coded, defining the player payoff functions as the objective functions to be maximized simultaneously. The body of the code sits within the command "`methods`". The code is then broken up into 3 subsections: Initialization, Calculation, and Sample Plotting.

3.2.2.1 Initialization

The Initialization section defines the domain of BSN1 where M stands for the number of objective functions and D stands for the number of decision variables. In the GUI, the user will assign M, and D to match the values coded in the MOOP they are looking to explore. The population size, N, will also need to be assigned by the user in the GUI. Lower and upper bounds are defined for all decision variables using the command `obj.Global.lower` and `obj.Global.upper`, respectively. The buyers offer exists on the interval $[0, 1]$ where 0 is the lower bound of the decision variable and 1 is the upper bound. Additionally, commission selection for a realtor looking for a quick sale exists on the interval $[0.05, 0.055]$, and commission selection for a realtor looking to maximize their commission exists on the interval $[0.055, 0.06]$. Finally, the encoding has been set to "`real`" as opposed to "`binary`" or "`permutation`", since we are exclusively dealing with a real-valued problem.

```
classdef BSN1 < PROBLEM
% <problem> <BSN1>
% Benchmark MOOP proposed by Gaudy, Gillman, and Schmitt

%------------------------------- Reference --------------------------------
% J. Gaudy, R. Gillman, and K. Schmitt, Buyer-Seller-Negotiator MOOP
% for evolutionary multiobjective optimization, The Negotiator's Role in a
% Buyer-Seller Game, 2019.
%------------------------------- Copyright --------------------------------
% Copyright (c) 2018-2019 BIMK Group. You are free to use the PlatEMO for
% research purposes. All publications which use this platform or any code
% in the platform should acknowledge the use of "PlatEMO" and reference "Ye
% Tian, Ran Cheng, Xingyi Zhang, and Yaochu Jin, PlatEMO: A MATLAB platform
% for evolutionary multi-objective optimization [educational forum], IEEE
% Computational Intelligence Magazine, 2017, 12(4): 73-87".
%--------------------------------------------------------------------------

    methods
        %% Initialization
        function obj = BSN1()
            obj.Global.M = 3;
            if isempty(obj.Global.D)
                obj.Global.D = 3;
            end
            obj.Global.lower    = [0,0.05,0.055];
            obj.Global.upper    = [1,0.055,0.06];
            obj.Global.encoding = 'real';
        end
```

Figure 3.7: Initialization section of the source code for BSN1

3.2.2.2 Calculation

Calculations of the objective functions succeeds initialization. `PopDec` refers to the population of the decision variables. This is a vector of values where each value is a 3-tuple, containing one value for each dimension of the decision space. The decision variables are the buyer's offer, `PopDec(:,1)`, the realtor's low commission selection, `PopDec(:,2)`, and the realtor's high commission selection, `PopDec(:,3)`. Population selection from the decision space occurs over a uniform distribution.

$$\texttt{PopDec} = \Big\langle (x_i, y_i, z_i),\ \ldots\ , (x_N, y_N, y_N) \Big\rangle, \quad i = 1, ..., N \tag{3.3}$$

where

$$\texttt{PopDec(:,1)} = (x_i, ..., x_N)$$

$$\texttt{PopDec(:,2)} = (y_i, ..., y_N)$$

$$\texttt{PopDec(:,3)} = (z_i, ..., z_N)$$

This vector is then applied to the objective functions, producing an $N \times 3$ number of rows is equal to the size of the population and the number of columns is equal to the number of objective functions. In the matrix `PopObj`, `PopObj(:,1)` references the seller's objective function, `PopObj(:,2)` the buyer's objective function, and `PopObj(:,3)` the realtor's objective function.

$$\texttt{PopObj} = \begin{pmatrix} f_1(x_i, y_i, z_i) & f_2(x_i, y_i, z_i) & f_3(x_i, y_i, z_i) \\ \vdots & \vdots & \vdots \\ f_1(x_N, y_N, z_N) & f_2(x_N, y_N, z_N) & f_3(x_N, y_N, z_N) \end{pmatrix}, \quad i = 1, ..., N \tag{3.4}$$

where

$$\texttt{PopObj(:,1)} = (f_1(x_i, y_i, z_i), ..., f_1(x_N, y_N, z_N))$$

$$\texttt{PopObj(:,2)} = (f_2(x_i, y_i, z_i), ..., f_2(x_N, y_N, z_N))$$

$$\texttt{PopObj(:,3)} = (f_3(x_i, y_i, z_i), ..., f_3(x_N, y_N, z_N))$$

```
%% Calculate objective values

%% In the following code, PobObj(:,1) represents the seller's objective function, PopObj(:,2) the buyer's
%% objective function, and PopObj(:,3) the realtor's objective function.  For the decision variables,
%% PopDec(:,1) represents the buyer's offer, PopDec(:,2) the realtor's low commission selections, and
%% PopDec(:,3) the realtor's high commission selection.

function PopObj = CalObj(obj,PopDec) %#ok<*INUSL>
    PopObj(:,1) = 0.5.*(PopDec(:,1) - (obj.Global.lower(:,1) + obj.Global.upper(:,1))/2. - ...
                  (PopDec(:,2).*PopDec(:,1) + PopDec(:,3).*PopDec(:,1))/2.) + ...
                  0.5.*((obj.Global.lower(:,1) + obj.Global.upper(:,1))/2.);
    PopObj(:,2) = 0.5.*((obj.Global.lower(:,1) + obj.Global.upper(:,1))/2. - PopDec(:,1)) + 0.5.*(0);
    PopObj(:,3) = 0.5.*((PopDec(:,2).*PopDec(:,1) + PopDec(:,3).*PopDec(:,1))/2) + 0.5.*(0);
end
```

Figure 3.8: Calculation section of the source code for BSN1

3.2.2.3 Sample Plotting

The final section of the code creates sample reference points on the Pareto front. A matrix of values containing all sample points for each objective function is constructed. Each column in `P` corresponds to the exact same column in `PopObj`. The exception here is that `PopDec` is replaced with an interval and step size for evenly spaced sample points within the interval of the decision variable based upon the size of the population. For instance,

```
(obj.Global.lower(:,1):(obj.Global.upper(:,1)-obj.Global.lower(:,1))/(N-1):obj.Global.upper(:,1))
```

creates a population of sample points in the interval of decision variable one from the low end to the high end with a step size equal to the range of the interval divided by one less the population size. Each decision variable, `PopDec`, is specified in this way for `P(:,1)`, `P(:,2)`, and `P(:,3)`. It is important to note that number of columns in `P` must match the number of columns in `PopObj`, as this value specifies the dimensions of the objective space.

```
        %% Sample reference points on Pareto front
        function P = PF(obj,N)
            P(:,1) = 0.5.*((obj.Global.lower(:,1):(obj.Global.upper(:,1)-obj.Global.lower(:,1))/(N-1):obj.Global.upper(:,1))' - ...
                      (obj.Global.lower(:,1) + obj.Global.upper(:,1))/2. - ((obj.Global.lower(:,2):(obj.Global.upper(:,2)- ...
                      obj.Global.lower(:,2))/(N-1):obj.Global.upper(:,2))'.*(obj.Global.lower(:,1):(obj.Global.upper(:,1)- ...
                      obj.Global.lower(:,1))/(N-1):obj.Global.upper(:,1))' + (obj.Global.lower(:,3):(obj.Global.upper(:,3)- ...
                      obj.Global.lower(:,3))/(N-1):obj.Global.upper(:,3))'.*(obj.Global.lower(:,1):(obj.Global.upper(:,1)- ...
                      obj.Global.lower(:,1))/(N-1):obj.Global.upper(:,1))')/2.) + 0.5.*((obj.Global.lower(:,1) + obj.Global.upper(:,1))/2.);
            P(:,2) = 0.5.*((obj.Global.lower(:,1) + obj.Global.upper(:,1))/2. - (obj.Global.lower(:,1):(obj.Global.upper(:,1)- ...
                      obj.Global.lower(:,1))/(N-1):obj.Global.upper(:,1))') + 0.5.*(0);
            P(:,3) = 0.5.*((obj.Global.lower(:,2):(obj.Global.upper(:,2)-obj.Global.lower(:,2))/(N-1):obj.Global.upper(:,2))'.* ...
                      (obj.Global.lower(:,1):(obj.Global.upper(:,1)-obj.Global.lower(:,1))/(N-1):obj.Global.upper(:,1))' + ...
                      (obj.Global.lower(:,3):(obj.Global.upper(:,3)-obj.Global.lower(:,3))/(N-1):obj.Global.upper(:,3))'.* ...
                      (obj.Global.lower(:,1):(obj.Global.upper(:,1)-obj.Global.lower(:,1))/(N-1):obj.Global.upper(:,1))')/2. + 0.5.*(0);
        end
    end
end
```

Figure 3.9: Sample plotting section of the source code for BSN1

Once coded, the GUI can be accessed and the algorithm to use, problem to solve, population size, number of objective functions, number of decision variables, and number of evaluations can be specified by the user. From here, results can be generated for the simultaneously maximized player utilities (see Figure 3.6).

Chapter 4

Experimental Results

4.1 Pareto Front

PlatEMO's test module was utilized and the appropriate values and options were selected. NSGA-II was selected for the algorithm dropdown, BSN1 was selected for the problem dropdown, the population size was initially set to 100, M (the number of objective functions) was set to 3, D (the number of decisions variables) was also set to 3, and evaluations was set to 10000 (see Figure 3.6). The resulting output appears to indicate the Pareto-optimal front is a straight line in objective space.

In Figure 4.1, the seller's range of payoffs can be seen along the f_1 axis, the buyer's along the f_2 axis, and the realtor's along the f_3 axis. We can interpret the Pareto front to indicate an increase in the realtor's payoff is a reflection of an increase in the seller's payoff. Also, as the payoff's increase for the seller and realtor, the buyer's payoff decreases. This is representative of typical market behavior between the three parties.

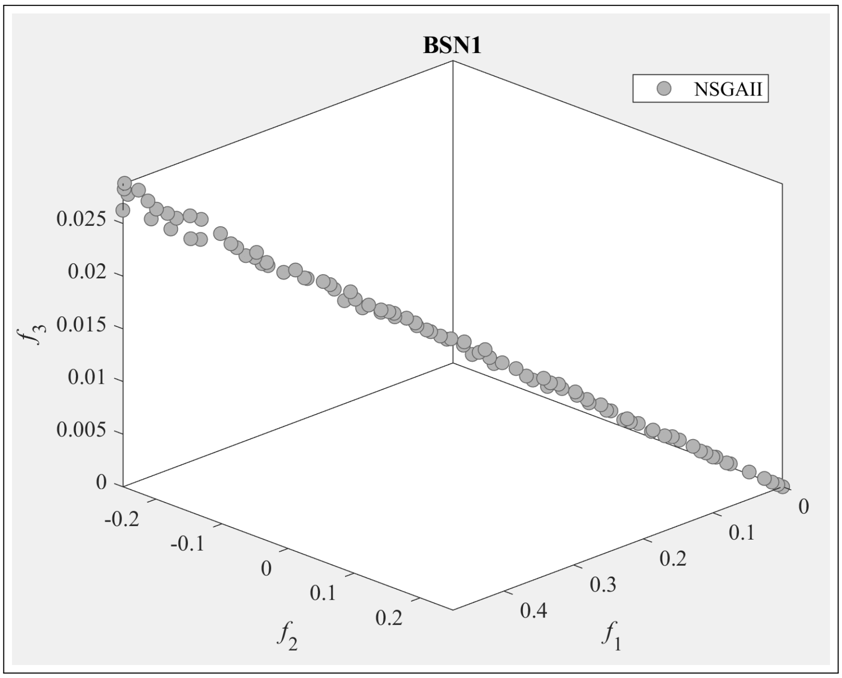

Figure 4.1: Pareto front for BSN1 under NSGA-II using a population size of 100

However, there appeared to be some scattering of solutions near the top of the front. To better explore this, the population size and number of evaluations were increased to 500 and 30000, respectively. This allowed for a larger sample to be drawn from the decision space and for triple the number of generations to be evaluated. The evaluating algorithm was also changed from NSGA-II to NSGA-III. The primary difference between these two algorithms is that NSGA-III uses reference points to maintain diversity among the solutions on the Pareto front. This allows for a more even distribution of points to better explore the objective space and the true Pareto front. NSGA-III works particularly well with three or more objective functions [19]. A comparison of how the two algorithms perform under the increased parameters can be see in Figure 4.2.

While the difference between the two algorithms is somewhat subtle, NSGA-III provides a more defined border to the Pareto-optimal front. It can now be

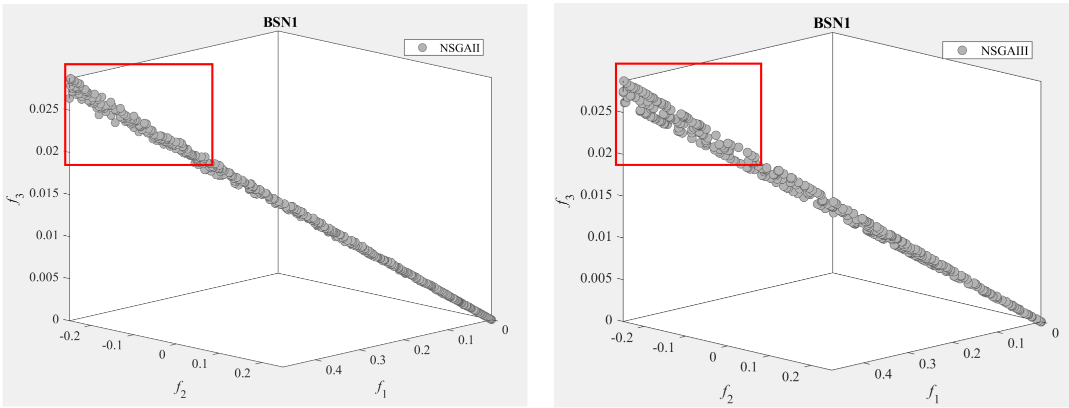

Figure 4.2: A comparison of the Pareto front in objective space using NSGA-II and NSGA-III

seen that the front, rather than a line, is a thin 2-dimensional pennant in the objective space. The influence in payoffs between the players still maintains the same behavior shown using NSGA-II with the smaller population size. To strictly look at the influence the realtor has on the seller's payoff, the objective space was rotated to show a 2-dimensional plane where the x-axis is the seller's payoff and the y-axis is the realtor's payoff. Figure 4.3 can be interpreted by stating that as the seller's payoff increases, the range of possible payoffs for the realtor increases. This could potentially be explained due to the existence of two types of realtor within the same objective function.

When interpreting player payoffs, it is important to remember that a probability of $\frac{1}{2}$ was applied to the player payoffs under the condition that the buyer's offer is greater than or equal to the seller's reserve price and $\frac{1}{2}$ for when it is less. This explains why objective space is truncated to values within the range $[0, 0.5]$ for the seller's payoff (f_1), $[-0.25, 0.25]$ for the buyer's payoff (f_2), and $[0, 0.03]$ for the realtor's payoff (f_3). The range for each of these payoffs is half the interval on which the decision variables lie. For instance, if the realtor chooses a commission of 6%, they have a 50% chance of achieving this commission and a 50% chance of gaining nothing depending on whether the buyer's offer is at very least equal

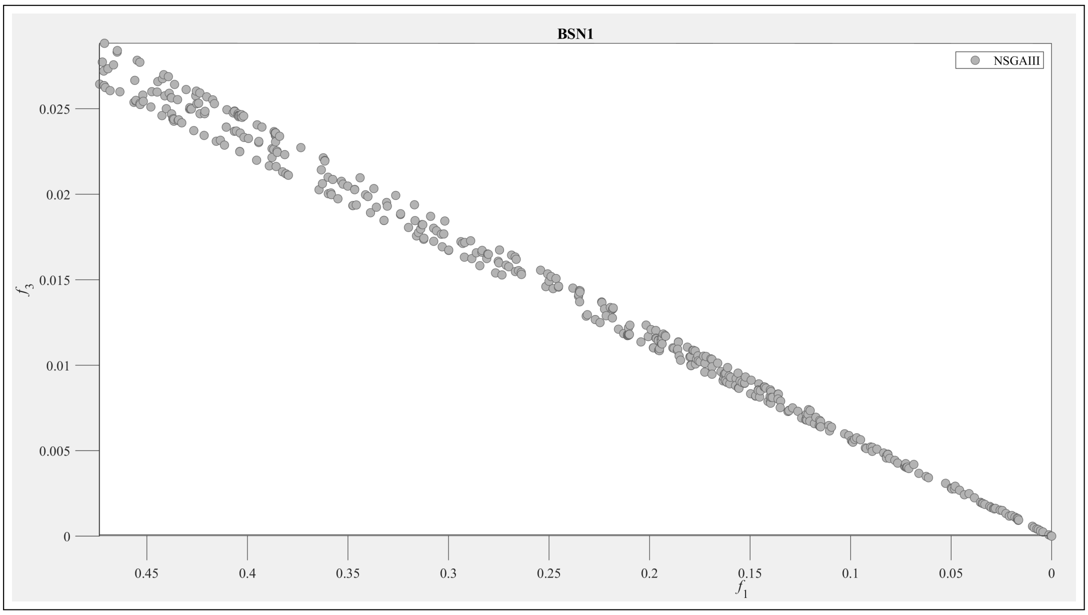

Figure 4.3: 2-dimension representation of the Pareto-optimal front when only considering the seller's and realtor's payoffs

to the minimum sale price. Therefore, the expected payoff for the realtor would result in a 3% commission of the buyer's offer.

It is also worth noting that the seller cannot receive a payoff less than 0, as he will always retain the value of his property in a scenario that the sale does not go through, even if that value is negligible. The realtor cannot receive a payoff less than 0 either, since they have not monetarily invested in the property outside of their time. However, the buyer can choose to make an offer over market value, although still in the fair market interval, in effort to meet the reserve price. This can be seen in Figures 4.1 and 4.2 as the buyer's payoff ranges on the interval $[-0.25, 0.25]$ as opposed to $[0, 0.5]$.

The *Pareto set* is the population on the Pareto-optimal front projected back to decision space. This allows the user to identify optimal values for the decision variables. Figure 4.4 shows the location of each member in the Pareto set in decision space. The buyer's offer selection can be seen along the f_1 axis, the realtor's low commission selection along the f_2 axis, and the realtor's high commission selection along the f_3 axis.

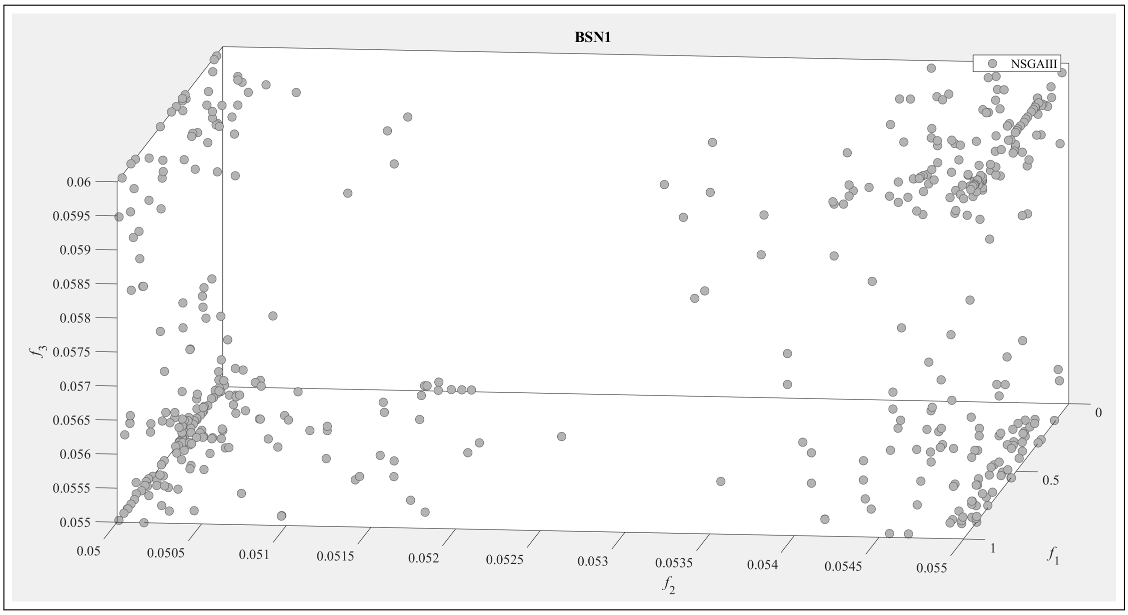

Figure 4.4: The Pareto set mapped to the decision space

From the plot of the decision space in Figure 4.4, it would appear optimality in player payoffs exists when f_2 and f_3 are maximized or minimized. This suggests that the realtor should select the the highest or lowest possible commission rate, but hardly between those two values. Since we are looking at two different types of realtors, it would make sense for a realtor looking to for a quick sale to always select the lowest possible commission rate and a realtor looking to maximize their profit to select the highest possible commission rate.

4.2 Comparison & Analysis

As was stated in Chapter 2, the results of the extensive game model indicated that it is within the realtor's best interest to align their strategy with that of the seller's. This is can be seen in the weak sequential equilibrium (equation 2.1). In a seller's market, the seller is assumed to be motivated, suggesting the realtor is better off to list the property for a quick sale; whereas, in a buyer's market, the seller is assumed to be indifferent, suggesting the realtor should list the property on the higher end of fair market.

The results from the Bayesian game model reiterate those from the extensive game model. Optimal payoff values exist on a line where the realtor's payoff increases in conjunction with the seller's payoff, and the buyer's payoff decreases accordingly. If we assume that a seller, whose payoff is closer to 0, values decreased market time exposure more than monetary gain, then this should be reflected in the realtor's commission selection. A realtor looking to maximize their commission in a market where the seller is motivated would result in a payoff outside the Pareto front displayed in Figures 4.1 and 4.2. Given the the Pareto-optimal front consists of all non-dominated payoffs within the objective space, it can be assumed that the Nash equilibrium for the Bayesian game is located somewhere on that front. However, the method for finding such a solution is outside the scope of this project.

Chapter 5

Conclusions

5.1 Application

The purpose of the models created for this study is to determine what influence a negotiator has on player payoffs in a buyer-seller game. When studying game theory, negotiators are rarely addressed/used when considering interactions between a buyer and seller. All negotiations, if any, are typically handled by the two engaged parties. By adding a vested third party interest to the game, it is made clear that player payoffs are dependent on how the acting third party aligns their strategy with those they are mediate for. Assuming alignment is made between the seller and negotiating party, the seller's final payoff, at very best, will still be less than if the third party were not considered, due to the required commission for the negotiator. This explains the rarity of negotiating parties in buyer-seller games.

While, this study focuses on the influence of a negotiator in relation to the real estate market, much of the findings from this research could be generalized to other such scenarios involving a vested negotiating party receiving some form of commission. Rationality from the models in this study suggests the realtor to only have a negative influence on player payoffs, particularly when divergent in their strategy selection with that of the party they are acting on behalf. By excluding the negotiator from a buyer-seller model, the two parties, while still working with imperfect information, can ensure their strategy is not being neglected by a negotiating self-interested party.

5.2 Future Study

Random selection for each of the decision variables in the Bayesian model is done using a uniform distribution. This is the generalized distribution for decision variables within the PlatEMO extension for MATLAB. However, when considering the realtor, it would make much more sense that there would be a skewed distribution to commission selection based on the type of realtor playing the game. A beta distribution where the mean, standard deviation, and skew, representative of the type of realtor, would need to be specified. This would allow for the distribution to define the type of realtor as opposed to the bounds on the interval.

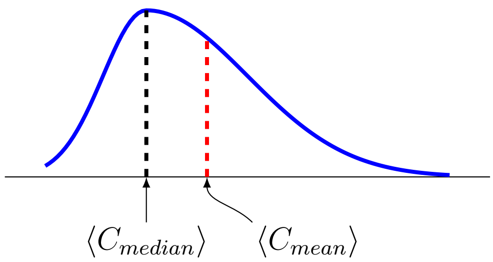

Figure 5.1: Right-skewed distribution representative of commission selection for a realtor looking to maximize their commission

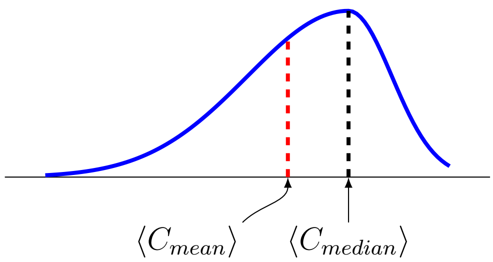

Figure 5.2: Left-skewed distribution representative of commission selection for a realtor looking for a quick sale

The realtor's commission, c, would exist on a singular interval where the range of the *max* and *min* value of the bounded interval are typical across various market climates. A realtor with a desire to maximize their commission would apply a right-skewed distribution to their commission selection where the mean would, most likely, exceed the median; whereas, a realtor looking for a quick sale would apply a left-skewed distribution to their commission selection where the median would, most likely, exceed the mean of the interval.

Expected payoff to the realtor could also be separated into two separate objective functions: one for a realtor looking to maximize their commission and the other for a realtor looking for a quick sale. This would alter the objective space from being 3-dimensional to 4-dimensional, Analysis and interpretation would require restricting the output to specific objective functions in order to determine the effects of one player on another.

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