Chapter 8: Insulators and Conductors

Chapter Learning Objectives: After completing this chapter the student will be able to:

- Explain how the polarization field in a dielectric material affects the electric field.
- Calculate electric flux density for a problem involving a dielectric material.
- Explain why there is no electric field inside a conductor.
- Use image charges to sketch and calculate the electric field lines near a conductor.

You can watch the video associated with this chapter at the following link:

Historical Perspective: Linus Pauling (1901-1994) was an American chemist and peace activist. His ground-breaking work on covalent and ionic chemical bonds earned him the Nobel Prize in Chemistry in 1954, and he also received the 1962 Nobel Peace Prize for his peace activism. He is one of only two people to win Nobel prizes in two different fields. The other is Marie Curie, who won for Physics in 1903 and Chemistry in 1911.

Photo Credit: [Public domain], via U.S. Library of Congress.
8.1 Introduction

Although we haven’t mentioned it before, all the work we have done until this point assumes that the charges, fields, forces, etc. are all in a vacuum. This means that there are no other messy particles around to interact with the charges and charge densities we are studying. Of course, this is not realistic. We will need to consider the effect of gases, liquids, and most importantly solids on our electric fields.

Solids break down into three main categories: Insulators, which do not conduct electricity, semiconductors, which conduct electricity a little bit (it’s complicated), and conductors, which conduct electricity like a champ. We will ignore semiconductors for this class, but both insulators and conductors deserve special attention.

8.2 Polarized Molecules and Dielectric Materials

Some molecules are very simple. Take argon, for example, As a noble gas, it doesn’t interact with any other atoms, and a molecule of argon is just a collection of perfectly symmetric argon atoms. But other materials, such as water, are much more complicated. As shown in Figure 8.1, when a water molecule is formed, the oxygen atom tends to pull electrons partially away from the hydrogen atoms. This makes one side of the molecule (with the oxygen atom) somewhat more negatively charged, while the other side (with the hydrogen atoms) is somewhat more positively charged.

![Figure 8.1. A polarized water molecule](image)

Such a molecule has the appearance of an electric dipole, which consists of a positive electric charge and an equal and opposite negative charge located nearby. To simplify our discussion, we will use the symbol in Figure 8.2 for an electric dipole.

![Figure 8.2. An electric dipole represented symbolically](image)
Many solid materials are composed of polarized atoms, which means that they are essentially composed of many, many electric dipoles. Typically, those dipoles are oriented randomly throughout the material, as shown in Figure 8.3.

![Figure 8.3. Solid material composed of randomly oriented electric dipoles](image)

(It’s not quite as simple as saying that the dipoles are randomly oriented, but it’s a fair assumption and a good approximation to reality for an introduction to this topic.)

When an electric field is applied to this solid material, the electric dipoles all respond by rotating to line up with the electric field, as shown in Figure 8.4.

![Figure 8.4. An electric field causes the polarized molecules to orient in the same direction](image)

The positive charges on the left edge and the negative charges on the right edge are intended to show the source of the electric field lines flowing through the material. (Again, it’s not quite this simple. They don’t all immediately snap into alignment. It depends on the strength of the electric field, material parameters, temperature, etc., but the concept shown is valid.)

If we now omit the external charges and electric fields from the figure (only for clarity—they are still there), we see in Figure 8.5 that the left edge of the material now has a surface charge composed of all negative charges, while the right edge of the material now has a surface charge composed of all positive charges. The middle of the material (shown in the dotted box) is
completely neutral. The net effect of this electric field has been to move a layer of negative charge to one edge and a layer of positive charge to the other edge.

**Figure 8.5.** Layers of surface charge form on both ends of the material.

Since the middle is neutral, we will omit it from the figure (again, just for clarity—it’s still there but it doesn’t affect the result). Figure 8.6 now shows that there is an induced electric field, which we will call the **Polarization Field** introduced into the interior of the solid material.

**Figure 8.6.** The Polarization Field appears.

Two observations about the polarization field:

1. It points in the opposite direction of the original electric field, reducing the effective electric field inside the solid material.
2. While it might seem that this polarization field is somewhat insignificant, it can actually have a massive impact on the results of the problem. As we will see, some highly polarizable materials can increase the effect of an external field by a factor of 1000 or more.

Of course, this discussion assumes that the electrons are firmly tied to their atoms, and that the atoms are essentially immobile. This is the case for an **insulator**. In short, an insulator has
essentially no free electrons, which means that it conducts essentially no current. You will also hear the word dielectric being used, and a dielectric is an insulator that also has highly polarized molecules, meaning that it does a great job of creating polarization fields. As we will see, polarizability is actually a very good thing, and so material scientists have been working a long time to discover/create dielectric materials that are highly polarizable.

8.3 Expanding Gauss’s Law

We will refer to the charge density on each surface of the material as \( \rho_p \), the surface charge density. The surface charge density is the source of the polarization field, and we can use the divergence to define the relationship between \( P \) and \( \rho_p \), as shown in Equation 8.1.

\[
\rho_p = -\nabla \cdot P
\]  

(Equation 8.1)

This relationship comes from Gauss’s law, which demonstrates how charge density is the source/sink (divergence) of an electric field.

We can now make a relatively small change to the work we have already done on electrostatics, and we will be able to include the effect of this polarization field in our results. This will start by defining a new quantity, called the Electric Flux Density (or, if you want to be old-school, the “Displacement Flux Density”). Represented by the variable \( D \), the electric flux density is defined as shown in Equation 8.2.

\[
D = \varepsilon_0 \cdot E + P
\]  

(Equation 8.2)

Notice that the electric flux density includes the original electric field, and it also includes the polarization field as well. We will see in just a little bit why the \( \varepsilon_0 \) is multiplying the \( E \) but not the \( P \).

Let’s reconsider Gauss’s Law, which we originally derived in Chapter 5 for the vacuum:

\[
\nabla \cdot E = \frac{\rho_v}{\varepsilon_0}
\]  

(Copy of Equation 5.6)

But now we know that we need to consider not only the charge density that is directly creating the electric field, but also the layer of surface charge density inside the dielectric material. This expansion of Gauss’s Law is shown in Equation 8.3.

\[
\nabla \cdot E = \frac{\rho_v + \rho_p}{\varepsilon_0}
\]  

(Equation 8.3)
Now, we will take the divergence of both sides of the definition of electric flux density (Equation 8.2):

\[ \nabla \cdot \mathbf{D} = \epsilon_0 \cdot \nabla \cdot \mathbf{E} + \nabla \cdot \mathbf{P} \]  
(Equation 8.4)

We will then substitute the definition of the surface charge density (Equation 8.1) and the expanded version of Gauss’s Law (Equation 8.3) into Equation 8.4.

\[ \nabla \cdot \mathbf{D} = \epsilon_0 \cdot \frac{\rho_v + \rho_p}{\epsilon_0} + \left(-\rho_p\right) \]  
(Equation 8.5)

A little bit of algebra gives us the final version of Gauss’s Law, which includes the effect of the surface charge density.

\[ \nabla \cdot \mathbf{D} = \rho_v \]  
(Equation 8.6)

This is the final version of Gauss’s Law. I promise not to make any other modifications to it for the rest of this course. It is also the first of the four of Maxwell’s Equations. From this one equation, we can derive all of electrostatics including the effect of dielectric materials.

As we have seen before, there is both a point form (Equation 8.6) and an integral form of each important equation in this course. The integral form of Gauss’s Law is shown in Equation 8.7.

\[ \oint_{\Delta s} \mathbf{D} \cdot d\mathbf{S} = Q_{enc} \]  
(Equation 8.7)

### 8.4 Dielectric Constant

We have made a lot of progress. We now know that we can simply use \( \mathbf{D} \) rather than \( \mathbf{E} \), and it will automatically take care of the effect of the polarization field \( \mathbf{P} \). But a problem still remains—we have no idea how strong the polarization field is for a given material. Recall that \( \mathbf{P} \) is an essential part of the definition of \( \mathbf{D} \), so we need to be able to calculate \( \mathbf{P} \) in order to find \( \mathbf{D} \).

\[ \mathbf{D} = \epsilon_0 \cdot \mathbf{E} + \mathbf{P} \]  
(Copy of Equation 8.2)

But there is good news. In many materials (and by many, I mean almost all materials), the strength of the polarization field is proportional to the applied electric field as long as the electric field is relatively small. (Of course, if the electric field is too strong, it will be powerful enough to actually move the atoms from their location or to rip electrons away from the molecules.) Assuming we’re not applying a super-strong electric field, we can confidently state that:
\[
P = \varepsilon_0 \chi_e E
\]

(Equation 8.8)

\(\chi_e\) is called the “electric susceptibility,” and it is a measure of how easily a material can be polarized by an external electric field. Don’t worry about memorizing this variable name, because we’re about to simplify things even further.

If we substitute our new knowledge of \(P\) from Equation 8.8 into Equation 8.2, we get:

\[
D = \varepsilon_0 \cdot E + \varepsilon_0 \chi_e E
\]

(Equation 8.9)

Algebra gives:

\[
D = \varepsilon_0 \cdot \left(1 + \chi_e\right)E
\]

(Equation 8.9)

We will now define the “relative dielectric constant” \(\varepsilon_r\) as \(1 + \chi_e\):

\[
\varepsilon_r = 1 + \chi_e
\]

(Equation 8.10)

We can now substitute this back into Equation 8.9:

\[
D = \varepsilon_0 \varepsilon_r E
\]

(Equation 8.11)

Thus, we see that the electric flux density \(D\) is proportional to the electric field \(E\).

But we can go one step further, combining the two constants into one:

\[
\varepsilon = \varepsilon_0 \varepsilon_r
\]

(Equation 8.12)

We will now have to be careful to refer to these three constants very precisely. The overall constant \(\varepsilon\) is the “dielectric constant,” \(\varepsilon_0\) is the “dielectric constant of free space,” and \(\varepsilon_r\) is the “relative dielectric constant. But this level of care if worthwhile, because we find the final version of the relationship between \(D\) and \(E\):

\[
D = \varepsilon E
\]

(Equation 8.13)
I have mentioned that there are four equations (Maxwell’s Equations) that can be used to derive all of electromagnetic field theory. This isn’t one of them, but it’s pretty close. There are three equations, called the Constitutive Equations, which support Maxwell’s Equations. This is one of the three constitutive equations. These are not fundamental laws of the universe, but they are extremely helpful relationships that explain how real materials tend to work. Just for reference, and as we will see in a couple of chapters, Ohms’ Law is also a constitutive equation.

The following table shows the relative dielectric constant for a wide range of materials:

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon_r$</th>
<th>Material</th>
<th>$\varepsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
<td>Glass</td>
<td>5-10</td>
</tr>
<tr>
<td>Air</td>
<td>1.0006</td>
<td>Porcelain</td>
<td>7</td>
</tr>
<tr>
<td>Paraffin</td>
<td>2</td>
<td>Rubber</td>
<td>7</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>Graphite</td>
<td>10-15</td>
</tr>
<tr>
<td>Oil</td>
<td>2.3</td>
<td>Silicon</td>
<td>11.68</td>
</tr>
<tr>
<td>Paper</td>
<td>3</td>
<td>Distilled Water</td>
<td>80</td>
</tr>
<tr>
<td>Mica</td>
<td>3-6</td>
<td>Titanium dioxide</td>
<td>86-173</td>
</tr>
<tr>
<td>Mylar</td>
<td>3.1</td>
<td>Strontium Titanate</td>
<td>310</td>
</tr>
<tr>
<td>Silicon Dioxide</td>
<td>3.9</td>
<td>Barium Titanate</td>
<td>1200</td>
</tr>
<tr>
<td>Fused Quartz</td>
<td>4</td>
<td>Calcium Copper Titanate</td>
<td>&gt;250,000</td>
</tr>
</tbody>
</table>

Example 8.1: Analyze the following system, in which two parallel metal plates, one charged to 10V and the other grounded, are placed 7mm apart. Calculate and sketch each of the quantities shown. If each metal plate is 1cm$^2$, how much total charge appears on that plate?
**Example 8.2:** Repeat example 8.1, but this time the space between the metal plates is filled with graphite ($\varepsilon_r=10$). Assume that there is an infinitesimal gap between the metal plates and the dielectric material.

![Diagram](image)

**Example 8.3:** Analyze the parallel plate system one last time, but this time there is a 1mm vacuum gap on each side of the 5mm of graphite.

![Diagram](image)
The most important thing to know about conductors is that they contain an essentially unlimited supply of free electrons, unattached to the atoms that “own” them, and these free electrons can move almost instantaneously in response to external forces. Since these electrons are free to move anywhere they want, they will always move toward locations where they will have the lowest potential energy. But once they get there, they make that location slightly less desirable for other electrons. This continues until every single point along the conductor is equally desirable/undesirable. Another way of saying this is that every point on a conductor will always be at the same voltage. (Remember, voltage is a representation of how much energy it would take to move a charge onto that point.) Alternatively, we can say that all conductors are equipotential surfaces.

But you already know that. When you connect a wire to a power supply, you assume that the voltage at the supply will be delivered to the breadboard you are connecting it to. (In reality, the wires we use are not perfect conductors, so the voltage delivered to the breadboard is just slightly less than the voltage coming out of the supply, but we are going to assume our conductors are perfect. It’s actually quite a reasonable assumption.)

Recall from chapter 7 that we derived a relationship between $E$ and $V$:

$$E = -\nabla V$$  \hspace{1cm} (Copy of Equation 7.31)

We also know that the gradient is the vector derivative of the voltage, so:

$$E = -\frac{\partial V}{\partial x} \mathbf{a}_x - \frac{\partial V}{\partial y} \mathbf{a}_y - \frac{\partial V}{\partial z} \mathbf{a}_z$$  \hspace{1cm} (Copy of Equation 7.30)

Since the voltage is constant at every point along the conductor, each of these partial derivatives will be zero, meaning that the electric field inside a conductor is always zero.

$$E = 0$$  \hspace{1cm} (Equation 8.14)

If electric field lines are aimed toward a conductor in equilibrium, then they must arrive at the conductor surface in a perpendicular direction. If there were a tangential component to the electric field in a perfect conductor, then the electrons would very quickly rearrange in response to that field until there is no longer a tangential component. Thus, electric fields must always be perpendicular to conductive surfaces.
But there is a problem. We have all of these highly developed equations and figures that show how electric fields look in the vicinity of charges and regions of charge density, and these are not guaranteed to be perpendicular to a conductor surface. Consider Figure 8.7, which would show electric field lines due to a point charge that have a tangential component to the conductor surface:

![Figure 8.7. Electric field lines incorrectly appear to have a tangential component](image)

So how can we solve this problem, other than modeling the nearly infinite number of free electrons in the conductor? It turns out that the Method of Images (or Method of Image Charges, or Method of Reflection) can be used to quickly and easily solve such problems. When there is a conductive surface near a charged object, the electric field lines behave exactly as if there were a mirror image charge that is equal and opposite the original charge reflected across the conductor. Of course, this image charge is not actually present, but the electric field lines bend just as they would if it were present.

Notice in Figure 8.8 how the field lines bend to become perpendicular to the conductor surface due to the (imaginary) image charge.

![Figure 8.8. With the image charge, all electric field lines are perpendicular to the conductor surface.](image)

In reality, the bottom half of this figure is all imaginary. Figure 8.9 shows what the real situation looks like. The bending of the electric field lines is actually due to free charges in the conductor surface that adjust themselves, thereby bending the lines.
Example 8.4: A point charge of 1C is located at (0,2), and there is a conductive plane on the x axis. What is the electric field at (3,0)?

8.6 Summary

- Polarized atoms rotate in response to an external electric field. We can account for this using electric flux density:
  \[
  \mathbf{D} = \varepsilon_0 \cdot \mathbf{E} + \mathbf{P}
  \]
- Gauss’s Law can be modified to include the effect of dielectric materials:
  \[
  \nabla \cdot \mathbf{D} = \rho_v
  \]
- The polarization field is typically proportional to the applied electric field, so electric flux density is also proportional to electric field:
  \[
  \mathbf{D} = \varepsilon \mathbf{E}
  \]
  \[
  \varepsilon = \varepsilon_0 \varepsilon_r
  \]
- The electric field is zero inside a conductor, and electric field lines must be perpendicular to the surface of a conductor.
- The method of images can be used to calculate the electric field in the vicinity of a conductor.